FX Option Backtester

User Guide

June 8, 2026

Abstract

This document specifies the design, methodology and quantitative behaviour of a modular FX option backtester applied to the EUR/NOK currency pair. Pricing relies on the Garman–Kohlhagen (1983) two-rate extension of Black–Scholes; risk is managed through a daily delta hedge of the option book against a spot position, and daily profit and loss is decomposed by Greek via a second-order Taylor expansion. The text covers the pricing model, data conventions, interpolation schemes, strategy life-cycle, hedge accounting, performance and risk metrics, and an empirical study of the delta-hedged long straddle over 2021–2024.

Contents

Notation

Symbol	Definition
S_t	spot exchange rate (NOK per EUR) at time t
K	strike of the option
T	time to maturity in years (act/365.25)
r_d	continuously-compounded domestic (NOK) interest rate
r_f	continuously-compounded foreign (EUR) interest rate
σ	implied volatility (decimal, annualised)
$N(\cdot)$	standard normal cumulative distribution function
$\phi(\cdot)$	standard normal probability density function
C, P	call and put prices in the domestic currency
Δ, Γ, ν, Θ, ρ_d, ρ_f	first-order Greeks of the option
F_t^T	forward outright rate at t for delivery at T

Table 1: Notation used throughout the document

---

1 Methodology

1.1 Pricing — Garman–Kohlhagen

Under the domestic risk-neutral measure $\mathbb{Q}_d$, the spot rate follows the two-rate geometric Brownian motion of Garman and Kohlhagen (1983),

$$\frac{dS_t}{S_t} \;=\; (r_d - r_f)\,dt + \sigma\,dW_t^{\mathbb{Q}_d} \tag{1}$$

where the drift carries the cost of carry $r_d - r_f$ that reflects the rate differential between the domestic and foreign yield curves. FX options are priced in the **domestic (quote) currency**; for EUR/NOK the quote currency is NOK, so all premia, P&L and Greeks are expressed in NOK.

The closed-form solution for European call and put prices reads

$$C = S_0\,e^{-r_f T}\,N(d_1) - K\,e^{-r_d T}\,N(d_2) \tag{2}$$

$$P = K\,e^{-r_d T}\,N(-d_2) - S_0\,e^{-r_f T}\,N(-d_1) \tag{3}$$

$$d_1 = \frac{\ln(S_0/K) + (r_d - r_f + \frac{1}{2}\sigma^2)\,T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T} \tag{4}$$

The full set of first-order Greeks delivered by the pricer is summarised below.

Greek	Call	Put
Δ	$e^{-r_f T}\, N(d_1)$	$-e^{-r_f T}\, N(-d_1)$
Γ	$\dfrac{e^{-r_f T}\, \phi(d_1)}{S_0\, \sigma \sqrt{T}}$	
ν (vega)	$S_0\, e^{-r_f T}\, \phi(d_1)\, \sqrt{T}$	
Θ	$-\dfrac{S_0\, e^{-r_f T}\, \phi(d_1)\, \sigma}{2\sqrt{T}} - r_d K e^{-r_d T} N(d_2) + r_f S_0 e^{-r_f T} N(d_1)$	$-\dfrac{S_0\, e^{-r_f T}\, \phi(d_1)\, \sigma}{2\sqrt{T}} + r_d K e^{-r_d T} N(-d_2) -$
ρ_d	$K T e^{-r_d T} N(d_2)$	$-K T e^{-r_d T} N(-d_2)$
ρ_f	$-S_0 T e^{-r_f T} N(d_1)$	$S_0 T e^{-r_f T} N(-d_1)$

Table 2: Closed-form Garman–Kohlhagen Greeks

Remark. In the implementation, vega is rescaled to a 1% absolute vol move ($\nu/100$) and theta is reported per calendar day ($\Theta/365$), to match trading-desk conventions and the units used in the attribution.

1.2 Market data and units

Dataset	File pattern	Unit normalisation
Spot	`*spot*.parquet`	NOK per EUR, used as-is
Forward points	`*forwardcurve*.parquet`	pips $\div$ 10 000; outright = spot + points
Deposit curves	`*depocurve*.parquet`	% $\to$ decimal; NOK = domestic, EUR = foreign
Vol surface	`*volatilitysurface*.parquet`	% $\to$ decimal

Table 3: Market data sources and normalisation conventions

Stacked frames are **pivoted once** into `{date: sub-frame}` maps so the day-by-day loop avoids repeated boolean masks.

1.3 Interpolation and extrapolation

Object	Interpolation	Extrapolation
Vol surface (strike)	cubic spline	linear, boundary-slope extension, floored > 0
Vol surface (time)	linear	linear
Forward and rate curves	linear	linear; duplicate tenors collapsed

Table 4: Interpolation and extrapolation scheme by object

1.4 Strategy life-cycle

1. **Roll scheduler** generates rebalancing dates (`W-FRI`, `MS`, `QS`, . . .).
2. On each roll date every **leg** is opened: its strike is resolved from the target delta (ATM maps to the forward outright; otherwise the GK delta is inverted on a dense strike grid).

3. The **sizing designer** (`Strategy.size`) converts the target notional into a per-leg traded notional (default: scale by the leg ratio).
4. Positions are **settled at intrinsic value** on expiry.

1.5 Daily delta hedging

Every business day the engine computes the **net option delta** (in foreign-currency units) and holds the opposite amount as a spot position. The hedge is rolled at the prevailing spot; its realised P&L is booked daily. Total daily P&L is therefore:

$$\mathrm{PnL}_{\mathrm{day}} = \underbrace{\Delta\,\mathrm{MTM}_{\mathrm{options}}}_{\text{option leg}} + \underbrace{\text{hedge PnL}}_{\text{spot leg}} \tag{5}$$

1.6 Performance and risk analytics

Let $\{r_t\}_{t=1}^{T}$ denote the daily strategy P&L series (annualisation factor $A = 252$). The engine reports the following option-aware metrics.

$$\begin{aligned}
\text{Sharpe ratio} &= \sqrt{A}\,\frac{\bar{r} - r_f/A}{\hat{\sigma}(r)} &(6)\\
\text{Sortino ratio} &= \sqrt{A}\,\frac{\bar{r} - r_f/A}{\hat{\sigma}_-(r)}, \quad \hat{\sigma}_-(r) = \sqrt{\tfrac{1}{T}\sum_t (\min(r_t, 0))^2} &(7)\\
\mathrm{VaR}_{1-\alpha} &= -\inf\{x : \mathbb{P}(r_t \le x) \ge \alpha\} &(8)\\
\mathrm{CVaR}_{1-\alpha} &= -\mathbb{E}\big[\, r_t \,\big|\, r_t \le -\mathrm{VaR}_{1-\alpha}\big] &(9)\\
\text{Max drawdown} &= \min_t \Big(C_t - \max_{s \le t} C_s \Big), \quad C_t = \sum_{u \le t} r_u &(10)
\end{aligned}$$

The default confidence level is $\alpha = 95\%$, and r_f is the risk-free benchmark (taken as zero in the empirical study below).

1.7 Greeks P&L attribution

The daily option P&L of legs alive on two consecutive days is decomposed via a second-order Taylor expansion around the previous day's spot, volatility and time,

$$\Delta P \;\approx\; \underbrace{\Delta\,\Delta S}_{\text{delta}} + \underbrace{\tfrac{1}{2}\,\Gamma\,(\Delta S)^2}_{\text{gamma}} + \underbrace{\nu\,\Delta\sigma}_{\text{vega}} + \underbrace{\Theta\,\Delta t}_{\text{theta}} + \underbrace{\varepsilon}_{\text{residual}} \tag{11}$$

The residual ε collects higher-order cross-Greeks (vanna $\partial\Delta/\partial\sigma$, volga $\partial\nu/\partial\sigma$), discretisation error and small inconsistencies in the volatility surface. Computing the decomposition **position-by-position over continuing legs** keeps premium cash flows on rolls out of the decomposition. The reported delta component is **net of the hedge P&L**: a perfect daily hedge in a frictionless setting drives this leg to zero, so it acts as a clean diagnostic of hedge slippage.

1.8 Signals

A signal maps the spot price history up to a roll date t onto a boolean (*trade* / *skip*). Two built-in filters are provided.

Momentum filter. With look-back L, the entry is allowed when the current spot exceeds its simple moving average,

$$\text{trade}_t \;\Leftrightarrow\; S_t \;\geq\; \frac{1}{L}\sum_{i=1}^{L} S_{t-i+1}$$

Realised-volatility filter. With look-back L and threshold $\bar{\sigma}$, the entry is allowed when the annualised realised volatility exceeds the threshold,

$$\text{trade}_t \;\Leftrightarrow\; \sqrt{A}\,\hat{\sigma}(\{r_{t-L+1},\ldots,r_t\}) \;\geq\; \bar{\sigma}$$

Filters compose by logical conjunction and are attached to any strategy via the `signal_fn` constructor argument.

2 Architecture

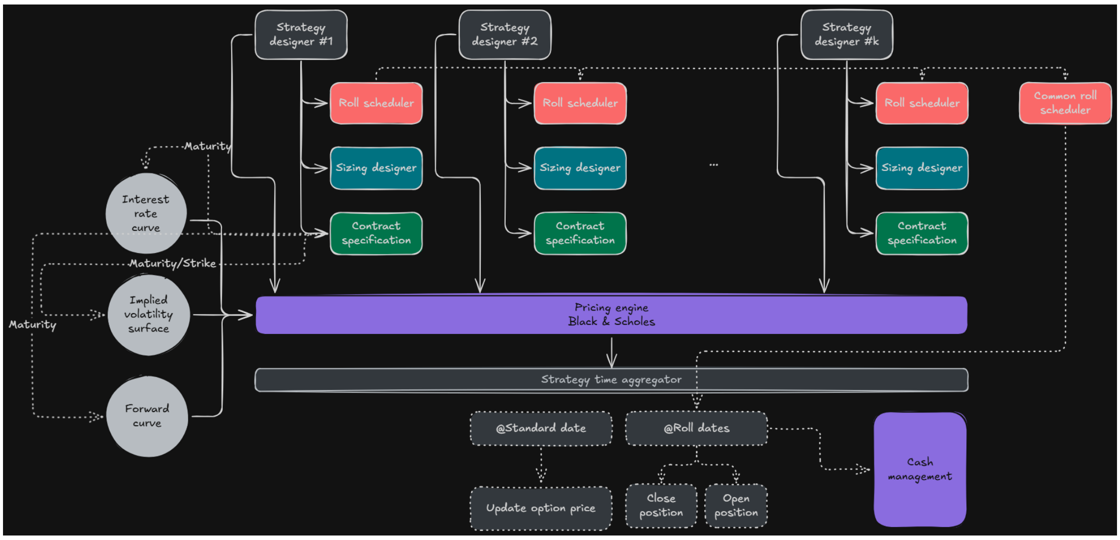

Figure 1: Component architecture of the FX option backtester

Listing 1: Package layout

```
src/
  data/loader.py          ingest + unit-normalise + pivot stacked
     Parquet
  curves/interpolator.py  linear curve interpolation/extrapolation
  volatility/surface.py   cubic-spline (strike) / linear (time) vol
     surface
  pricing/                Garman-Kohlhagen price + Greeks (scalar and
     vectorised)
  strategies/             Straddle, CallRatioSpread, CalendarSpread,
     signals, sizing
  portfolio/position.py   option positions, cash, spot delta-hedge book
  analytics/              performance metrics + Greeks P&L attribution
  engine.py               orchestrator: roll -> price -> hedge -> MTM ->
      attribute
notebooks/backtester.ipynb   end-to-end walkthrough
outputs/                     charts and CSV results (generated by
   notebook)
```

Per-strategy **roll scheduler**, **sizing designer** and **contract specification** feed a shared Garman–Kohlhagen pricing engine, whose output is consumed by a time aggregator producing standard-date MTM updates and roll-date open/close events with cash management.

3 How to use

3.1 Installation

```
poetry install
# or without Poetry:
pip install pandas numpy scipy matplotlib seaborn pyarrow jupyter
```

Place the four Parquet market data files in `data/`.

3.2 Run a backtest

```
from src.engine import Backtester
from src.strategies.straddle import Straddle

bt = Backtester("data")
bt.load_data()

strategy = Straddle(tenor_days=30, direction=1)
results = bt.run(
    strategy,
    start="2021-01-01",
    end="2024-12-31",
    notional=1_000_000,
    roll_freq="W-FRI",
)

print(results["metrics"])
results["daily_pnl"].cumsum().plot()
```

`results` keys: `metrics`, `daily_pnl`, `components` (option vs. hedge P&L), `greeks`, `attribution`, `portfolio`.

3.3 Add a signal

```
from src.strategies.signals import momentum_signal

strategy = Straddle(
    tenor_days=30,
    signal_fn=lambda h: momentum_signal(h, lookback=20),
)
```

3.4 Notebook and tests

- End-to-end walkthrough: `notebooks/backtester.ipynb`
- Unit tests: `poetry run pytest`
  Covers pricing parity, Greek signs, curve and vol interpolation, performance and attribution analytics.

4 Quantitative deep dive — delta-hedged long straddle

Setup. Long 30-day ATM straddle on EUR/NOK, rolled weekly, delta-hedged daily, January 2021 to December 2024, notional 1 000 000 NOK.

Economic rationale. Once delta is neutralised, the straddle is a pure bet on **realised vs. implied volatility**. Its carry is approximately

$$\text{PnL} \approx \tfrac{1}{2}\,\Gamma\, S^2 \left(\sigma^2_{\text{realised}} - \sigma^2_{\text{implied}}\right) dt \tag{12}$$

The strategy harvests gamma when spot moves exceed the implied volatility paid, and bleeds theta when they do not.

Backtest results.

Metric	Always-on	Momentum-gated
Total PnL (NOK)	$\approx 1.30\,\text{M}$	$\approx 1.68\,\text{M}$
Sharpe ratio	0.13	0.23
Sortino ratio	0.15	0.24
Max drawdown	$\approx -2.87\,\text{M}$	$\approx -1.61\,\text{M}$

Table 5: Performance comparison: always-on vs. momentum-gated straddle

The **Greeks attribution** confirms the textbook long-gamma signature: cumulative gamma $\approx +21\,\text{M}$ is almost entirely offset by theta $\approx -24\,\text{M}$, with hedged-delta, vega and residual all small.

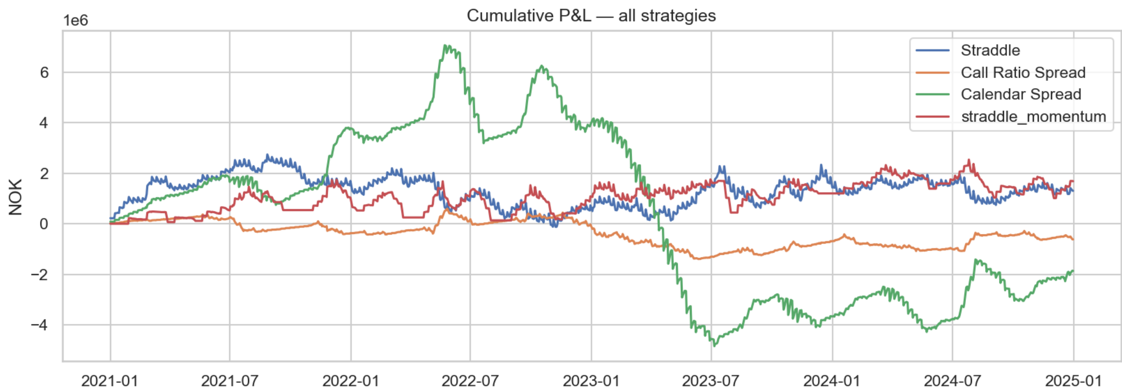

Figure 2: Cumulative P&L of the three multi-leg strategies and the momentum-gated straddle over the backtest window

Read-through to EUR/NOK. NOK is an oil-sensitive, lower-liquidity currency whose volatility clusters in risk-off episodes (e.g. 2022). The gamma harvest dominates during those bursts and decays in calm, range-bound regimes. A simple momentum filter removes some low-vol roll periods, roughly **doubling the Sharpe ratio and halving the maximum drawdown** while preserving gross P&L — a favourable risk-adjusted outcome.

Limitations. Mid-price execution (no bid/ask spread or hedging transaction cost), discrete daily hedging (gamma slippage between rebalances), and a Taylor attribution that omits higher-order cross-Greeks (vanna, volga), visible as the residual term.

Extensions. Natural follow-ups include (i) modelling bid/ask and hedge transaction costs as a haircut on the daily P&L, (ii) incorporating stochastic-volatility corrections (vanna/volga adjustment), (iii) calibrating an intraday rehedging frequency to bound gamma slippage, (iv) extending the signal library with macroeconomic or risk-on/risk-off filters.

References

- Garman, M. B. and Kohlhagen, S. W. (1983). *Foreign currency option values.* Journal of International Money and Finance, 2(3), 231–237.
- Hull, J. C. (2018). *Options, Futures, and Other Derivatives*, 10th edition. Pearson.
- Reiswich, D. and Wystup, U. (2010). *A guide to FX options quoting conventions.* Journal of Derivatives, 18(2), 58–68.
- Wilmott, P. (2007). *Paul Wilmott on Quantitative Finance*, 2nd edition. John Wiley & Sons.
- Bergomi, L. (2016). *Stochastic Volatility Modeling.* Chapman & Hall.