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MASTER'S THESIS

The Total Portfolio Approach Tested Against Market Regimes

An Analysis of the Opportunity Cost for an Institutional Insurer

Presented and defended by

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Abstract

The Total Portfolio Approach (TPA) has gradually established itself among several large institutional investors as a way of managing a portfolio as a whole rather than through separate asset classes. Within this logic, each investment is assessed relative to the other opportunities available within the portfolio.

Yet one question is rarely addressed: does this opportunity cost remain the same when the market environment changes?

To answer this question, this thesis relies on a Hidden Markov Model (HMM) to identify four macro-financial regimes (Goldilocks, overheating, recession and stagflation) over the period from February 2004 to May 2026. The analysis focuses on five factors representative of the main sources of risk of a multi-asset portfolio.

The results show that the ranking of factors varies strongly from one regime to another. The Equity factor, for example, is the most attractive in the Goldilocks regime, but becomes the least attractive during recessions. Conversely, the Duration factor follows an opposite pattern. Likewise, the relationships between assets do not remain constant: the correlation between equities and bonds rises from 0.38 in favorable periods to 0.70 in stagflation, which noticeably reduces the benefits of diversification.

These results show that it is difficult to summarize the entire economic cycle with a single opportunity cost. The TPA framework remains relevant, but its application benefits from taking the market context into account. When regimes change, assets do not behave in the same way, and their relative attractiveness can shift considerably. The idea of a TPA adjusted to market regimes therefore makes it possible to preserve the logic of the total portfolio while adapting it more closely to observed conditions.

Keywords:

Total Portfolio Approach; opportunity cost; market regimes; Hidden Markov Model; asset allocation; institutional investor; diversification.

Abstract

The Total Portfolio Approach (TPA) has become increasingly popular among large institutional investors to manage portfolios from a total-portfolio perspective rather than through separate asset-class silos. Within this framework, investment decisions are based on a common opportunity cost. However, little attention has been paid to a simple question: does this opportunity cost remain stable across different market environments?

This thesis addresses that question using a Hidden Markov Model (HMM) to identify four macro-financial regimes: Goldilocks, Overheating, Recession and Stagflation, over the period from February 2004 to May 2026. The analysis relies on five multi-asset risk factors designed to capture the main drivers of portfolio returns.

The results indicate that asset attractiveness changes substantially across regimes. The Equity factor ranks first during Goldilocks periods but falls to the bottom of the ranking in recessions. The Duration factor follows the opposite pattern. In addition, the correlation between equities and bonds rises from 0.38 in favorable environments to 0.70 during stagflation, reducing the diversification benefits usually expected by long-term investors.

These findings suggest that relying on a single opportunity-cost framework may overlook important differences across market regimes. Rather than challenging the foundations of the TPA, the results support a regime-aware implementation in which investment opportunities are assessed according to the prevailing economic environment.

Keywords:

Total Portfolio Approach; opportunity cost; market regimes; Hidden Markov Model; asset allocation; institutional investor; conditional diversification.

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List of Abbreviations

Acronym	Meaning
TPA	Total Portfolio Approach
SAA	Strategic Asset Allocation
HMM	Hidden Markov Model
SCR	Solvency Capital Requirement
ALM	Asset-Liability Management
MVO	Mean-Variance Optimization
ERC	Equal Risk Contribution
OC	Opportunity Cost
EM	Expectation-Maximization
OOS	Out-Of-Sample
NBIM	Norges Bank Investment Management
CPP	Canada Pension Plan

General Introduction

Context and Stakes

For about a decade now, institutional investors have been operating in an environment profoundly different from the one that shaped traditional allocation models. The long period of very low interest rates, followed by the marked return of inflation from 2022 onwards, together with the growing role played by private assets, have progressively called into question the decision frameworks inherited from the 1990s.

Faced with these developments, several large international investors, such as Australia's Future Fund, CPP Investments or Norges Bank Investment Management, have adopted the Total Portfolio Approach (TPA). The principle of the TPA is to reason at the level of the portfolio as a whole, rather than managing each asset class separately. An investment is therefore not only compared with similar assets, but assessed according to its impact on the overall portfolio.

This approach is of obvious interest to large investors, as it makes it possible to better connect allocation decisions with one another. It nonetheless remains difficult to apply in practice. One of the sensitive points concerns the opportunity cost used to compare investments: it is often treated as though it remained relatively stable over time, even though market conditions can change sharply. In other words, it assumes that the criteria used to compare and rank investments remain broadly the same regardless of changes in the economic and financial environment. It is precisely this assumption that this thesis sets out to examine.

If we accept that the ranking of assets by marginal contribution to the total portfolio varies strongly according to the macroeconomic regime (Goldilocks, overheating, recession or stagflation), then the static SAA loses part of its operational relevance. It is precisely this tension that this thesis sets out to examine, drawing on the data and tools developed during an apprenticeship at the Investment Department of Generali France.

Research Problem

The literature devoted to the TPA is abundant but largely produced by practitioners (asset managers, consultants, index providers). It describes the “what” and the “why” of the TPA, more rarely the quantitative “how,” and almost never the sensitivity of the framework to market conditions. The research question of this thesis is the following:

Is the ranking of assets by opportunity cost, which underpins the allocation decision in the Total Portfolio Approach, stable over time, or does it reverse across market regimes; and, in the latter case, does a regime-conditional TPA improve the consistency of decisions relative to a static TPA?

This thesis seeks to answer three questions: do distinct and identifiable macro-financial regimes exist? Do these regimes modify the opportunity cost and the ranking of assets? Finally, do they influence diversification, a central element of the Total Portfolio Approach?

Positioning and Contributions

This thesis does not seek to propose a new theory of the TPA. Its objective is more targeted: to test empirically the assumption of stability of the opportunity cost. To this end, the analysis draws on more than twenty years of market data and on a Hidden Markov Model that makes it possible to identify different macro-financial regimes.

The main contribution is therefore to study how the relative attractiveness of risk factors evolves across regimes, and then to discuss what these results imply for the implementation of the TPA in an institutional setting.

Structure of the Thesis

The thesis is organized into four parts. The first two chapters present the professional context, the insurance framework and the investment constraints specific to an insurer. Chapters 3 and 4 review the foundations of the Total Portfolio Approach and the notion of opportunity cost. Chapter 5 details the methodology adopted, in particular the construction of the factors and the identification of regimes using a Hidden Markov Model. Finally, Chapters 6 and 7 present the empirical results, their implications and the limitations of the analysis.

Scientific Approach

The approach followed consists of starting from a simple hypothesis — "the stability of the opportunity cost" — and then confronting it with the data. If the characteristics of the risk factors vary according to market regimes, then their ranking should evolve as well.

The objective is not to demonstrate that a conditional approach systematically outperforms a static approach. It is rather to check whether the stability assumption, often implicit in the TPA, remains defensible once changes in market regime are taken into account.

Chapter 1 — Professional Context: Generali and Investment Management

1.1 The Generali Group

Founded in Trieste in 1831, Assicurazioni Generali is today one of the world's leading insurance and asset management groups. Its activity is organized around three main businesses: life insurance, property and casualty insurance, and asset management. For a group of this size, investment decisions are central, as they must make it possible to meet the commitments made to policyholders, in particular on life insurance contracts, which are often long-term.

Unlike a traditional asset manager, an insurer does not seek only financial performance. It must also ensure that its portfolio remains consistent with its future commitments, their horizon, their sensitivity to interest rates and the regulatory constraints that accompany them.

The apprenticeship behind this thesis took place within the Investment Department, whose mission is to define and implement the strategic and tactical allocation of the general portfolio, in line with the Group's risk appetite and regulatory capital constraints. It is within this framework that the reflection on the Total Portfolio Approach took shape, as a possible alternative to a strategic allocation still largely organized in silos.

1.2 Assignments Carried Out During the Apprenticeship

The work carried out during the apprenticeship was mainly part of an effort to develop tools for monitoring, steering and risk analysis for the investment portfolios.

Several structuring projects were carried out:

- The implementation of an automated pipeline for the valuation and steering of the Growth (Croissance) portfolios, making it possible to centralize market data, automate performance and risk calculations, and make the portfolio monitoring processes more reliable.
- The development of a Streamlit dashboard dedicated to monitoring hedging positions, connected to Bloomberg data in real time. This tool integrates both a detailed view of derivative instruments, market indicators and macroeconomic data in order to improve the monitoring of exposures and support decision-making.
- The replication of market sentiment indicators, in particular the Goldman Sachs Risk Appetite Indicator (GSRAI) and Société Générale's SG Risk Indicator. The objective was to have a synthetic reading of the market environment in order to identify “risk-on” and “risk-off” phases.

- The replication of the paper presented by Bouyé and Teïletche, "Regime-based Strategic Asset Allocation" (2025), in order to study the contribution of regime models to strategic allocation decisions and to assess their relevance in an institutional management context.[1]
- The preparation of presentations for market meetings, devoted to topical themes related to finance and new technologies. Among the subjects studied were, in particular, the rise of stablecoins, the possible evolutions of international payment infrastructures, as well as the potential applications of quantum computing to financial problems such as optimization, risk management and asset valuation.

Beyond their operational dimension, these various projects helped to feed a broader reflection on the influence of the market environment on investment decisions. The research carried out on market regimes, risk indicators and allocation models was in particular a direct source of inspiration for the research problem developed in this thesis.

1.3 From the Operational Tool to the Research Question

The interest of this thesis lies in the link between the work carried out in the company and the academic reflection it gave rise to. Among the various projects conducted during the apprenticeship, the work on market regime detection particularly caught my attention. Observing the results obtained, it appeared that the relative attractiveness of the different risk factors varied strongly according to the economic and financial environment.

This observation quickly raised a question: if market regimes influence asset characteristics so much, can the opportunity cost, a central notion of the Total Portfolio Approach, really be considered stable over time? It is this question, born of an operational need and then nourished by the academic literature, that gradually gave rise to the research problem developed in this thesis.

1.4 The Investment Department: Organization and Processes

Within an insurer such as Generali, investment decisions involve several levels of decision-making. The broad orientations defined by senior management must then be translated into allocation choices and then into concrete management decisions, while respecting the constraints specific to the insurance sector, particularly in terms of solvency, liquidity and risk management.

In this context, quantitative analyses take on an important role. It is no longer just a matter of monitoring the portfolio by broad asset classes, but also of identifying the factors that truly explain risks and performance. Two assets belonging to the same class may react differently to market conditions, while very different assets may share a common exposure.

Amundi Asset Management, Multi-Asset Total Portfolio Approach in a Complex World (Amundi, 2020).[1]

The projects carried out during the apprenticeship went in this direction. The development of market sentiment indicators, the analysis of macro-financial regimes and the construction of steering tools all aimed to improve the reading of investment risks and opportunities.

More broadly, this experience showed the value of reasoning in terms of risk factors rather than solely in terms of asset classes.

This view makes it possible to compare very different investments on a common basis and connects directly with the philosophy of the Total Portfolio Approach, which seeks to assess each decision at the scale of the portfolio as a whole.

Chapter 2 — The Investment Framework of an Insurer Under Solvency II

2.1 Specificities of Insurance Investment

For an insurer, investing does not consist solely of seeking the best possible performance. The assets held are first and foremost intended to finance the commitments made to policyholders. Investment decisions must therefore take into account many parameters, such as the sensitivity of liabilities to interest rates, their long-term horizon or liquidity needs.[2]

The Solvency II regulation bears directly on these decisions. It requires the insurer to set aside a solvency capital (the SCR), calibrated to absorb a shock that would in principle only be seen once every two hundred years. As the required capital depends on the risk of the assets, equities and real estate consume more own funds than sovereign bonds or certain credit exposures. This is one of the reasons why the portfolios of life insurers remain predominantly bond-based.

Liquidity needs linked to surrenders and benefit payments also limit the place of less liquid assets. For an insurer, the opportunity cost therefore cannot be reduced to the expected return: it must also take into account the capital consumed and the constraints that each asset adds to the portfolio.

This specificity is particularly important within the framework of this thesis. Certain market environments can simultaneously degrade the performance of risky assets and the effectiveness of hedging assets, making portfolio management more complex. As we will see later, periods of stagflation or of strong tensions on interest rates are among the regimes in which the differences between risk factors change most strongly, both in terms of performance and of correlation.

[2] EIOPA, Guidelines on the Valuation of Technical Provisions — Solvency II (European Insurance and Occupational Pensions Authority, 2015).

2.2 The SCR as a Structuring Constraint

Concretely, the SCR is calculated by applying shocks to each source of risk. In the standard formula, listed equities undergo a shock of the order of 39% (adjusted by a symmetric adjustment depending on the phase of the cycle), whereas sovereign bonds of the European Economic Area carry very little, if any, capital charge. Credit, for its part, is penalized according to its duration and the quality of the issuer. Two assets showing the same expected return therefore do not have the same capital cost: it is this regulatory charge, as much as the return-risk trade-off, that guides an insurer's allocation. A volatile asset such as equities must in a sense “pay” its capital cost to justify its place in the portfolio, which connects directly to the logic of opportunity cost at the heart of this thesis.

Three ideas emerge from this analysis. First, the long-term investment horizon specific to insurers makes particularly attractive the idea of a relatively stable opportunity cost, which would serve as a reference for allocation decisions. This is indeed one of the foundations of the Total Portfolio Approach.

However, observation of the markets shows that this stability is far from guaranteed. The relative performance of assets and the benefits of diversification evolve noticeably according to economic phases, in particular during periods of stress.

Consequently, the challenge is perhaps not to oppose a long-term view to the realities of market regimes, but to combine them. The analysis conducted in this thesis is part of this perspective: preserving the principles of the TPA while taking market conditions into account.

2.3 Implications for Strategic Allocation

An insurer's investment horizon is particularly long, which can justify the use of a relatively stable opportunity cost to guide allocation decisions. This idea is at the heart of the Total Portfolio Approach. However, observation of the markets shows that the characteristics of assets can evolve strongly according to the economic environment. Returns, risks and correlations are not the same in a period of growth as in a period of recession or stagflation.

This observation leads to questioning the way in which the TPA should be implemented. Rather than calling its principles into question, it seems relevant to examine to what extent taking market regimes into account can enrich the analysis of the opportunity cost. It is this question that guides the remainder of the thesis.

2.4 Asset-Liability Management and Investment Horizon

Life insurance is characterized by a very long investment horizon, with commitments to policyholders potentially extending over several decades. This specificity allows insurers to invest in assets offering long-term return potential, including certain less liquid assets.[3]

However, this long-term view must contend with more immediate constraints. Portfolios are monitored continuously and policyholders retain the possibility of making surrenders, in particular when market conditions become less favorable. In certain stress scenarios, an insurer may thus be confronted simultaneously with a fall in the value of its assets and a rise in liquidity needs.

This reality is directly linked to the research problem of this thesis. The Total Portfolio Approach was designed for investors able to adopt a very long perspective and to withstand market fluctuations without particular constraint. For an insurer, the situation is more complex: changes in economic regime influence not only the performance of assets, but also the constraints it faces. In this context, taking market regimes into account appears less as a methodological refinement than as a necessity to inform allocation decisions.

2.5 Profit-Sharing and the Non-Linearity of Liabilities

In life insurance, policyholders may benefit from part of the financial performance of the portfolio, while certain contracts also provide for minimum guarantees. For the insurer, a lasting fall in returns therefore does not have the same effects as a rise in the markets. This makes management more sensitive to the financial context.

This point is important for interpreting the results of the thesis. Even if the empirical analysis focuses on assets, market regimes can also modify the insurer's commitments. The effects observed on the asset side must therefore be placed within a broader asset-liability management logic. For reasons of scope and data availability, the empirical analysis focuses on assets, but the observed effects must be interpreted in this broader context. The variation of the opportunity cost across market regimes is therefore likely to entail even more significant consequences when considered jointly with the specificities of insurance liabilities.

[3] Andrew Ang, "Total Portfolio Approach for Long-Term Investors," Journal of Financial Markets, 2022; John Y. Campbell and Luis M. Viceira, Strategic Asset Allocation: Portfolio Choice for Long-Term Investors (Oxford University Press, 2002).

Chapter 3 — Theoretical Foundations of the Total Portfolio Approach

3.1 Origins of the Total Portfolio Approach

The Total Portfolio Approach (TPA) developed over the course of the 2010s among several large institutional investors, in particular sovereign wealth funds and pension funds. It addresses a fairly classic limitation of traditional allocation: the portfolio is often divided into asset classes managed separately, whereas risks sometimes overlap from one bucket to another.

The principle of the TPA is to start from the portfolio as a whole. An investment is therefore not only compared with the assets of its own category; it is assessed according to its effect on the overall portfolio, in particular on the return-risk trade-off.

This approach rests on a few simple ideas: reducing management in silos, better identifying common risk factors, and comparing investments on the basis of a common opportunity cost. It is above all this last point that concerns this thesis, as it raises an important question: does this opportunity cost remain stable when the market environment changes?

3.2 The Practitioner Literature: A Widely Shared Consensus

The literature on the TPA is mainly carried by market players: management firms, consultants and data providers. Even if each approaches it from a different angle, the same ideas often recur: reasoning better at the scale of the overall portfolio, going beyond asset-class silos, and improving decision-making.

AQR, for example, draws the TPA closer to risk-factor management, while MSCI insists more on the tools needed to measure portfolio exposures in a cross-sectional manner. Brookfield, Franklin Templeton or Amundi place more emphasis on issues of governance, organization and implementation. This literature is therefore useful for understanding why the TPA attracts institutional investors, but it generally remains less precise on the way the framework reacts to changes in the market environment.[4]

Overall, these works are mainly interested in how to deploy the TPA and in the advantages it can bring. On the other hand, they rarely address the question of the stability of the opportunity cost over time. This dimension nonetheless appears important once the characteristics of markets evolve according to

[4] Amundi Asset Management, Multi-Asset Total Portfolio Approach in a Complex World; AQR Capital Management, Total Portfolio Approach, Alternative Thinking (AQR Capital Management, 2023), https://www.aqr.com; Brookfield Asset Management, The Emergence of Total Portfolio Approach Investing (Brookfield Asset Management, 2026).

economic phases. Finally, the works of EDHEC provide a more academic contribution, in particular on the integration of private assets within an overall-portfolio logic.[5]

Overall, this literature is mainly interested in the benefits of the TPA and in the conditions of its deployment. It explains why this approach is attractive and how investors can adopt it. On the other hand, one question is rarely addressed: that of its robustness to changes in market regime.

Most analyses implicitly rest on a long-term view in which the characteristics of assets are considered relatively stable. Assumptions about return, volatility or correlation are generally treated as background parameters, even though market experience shows that they can evolve strongly according to the economic context. This lack of reflection on the variability of the opportunity cost over time is one of the main starting points of this thesis.

Source (category)	**Main contribution**	**Regime dependence**
AQR (practitioner)	TPA as an extension of factor investing	Implicit, untested
MSCI (practitioner)	Cross-sectional factor attribution	Absent
EDHEC (academic)	TPA and private assets, illiquidity	Partial (liquidity)
Brookfield / Franklin	Governance and rise of the TPA	Absent
Amundi (practitioner)	Multi-asset implementation	Mentioned, not formalized
Quantitative literature	General formalization of the TPA	Static framework

Table 3.1 — Main contributions of the literature devoted to the Total Portfolio Approach

3.3 The Academic Critique: A Genuine Innovation?

Despite its success with many institutional investors, the Total Portfolio Approach does not achieve unanimity. Some authors consider that it is above all a new way of presenting concepts already well known in portfolio management. Indeed, several of its principles are close to older approaches, such as factor management, risk budgeting or the modern portfolio theory developed by Markowitz.[6]

The question is also whether the TPA really changes investment decisions. Does it lead to allocations different from those of a well-constructed SAA, or is it above all a new way of formalizing existing practices? For now, the empirical evidence remains limited. The debate therefore bears more on the management philosophy and the organization of teams than on a quantified comparison of performance.

This question is of twofold interest. On the one hand, it can be studied empirically from market data. On the other, it does not call into question the logic of the TPA itself. Showing that the opportunity cost varies

[5] EDHEC-Risk Institute, Total Portfolio Approach and Private Assets, Part 1 (EDHEC Business School, 2023); Part 2 (EDHEC Business School, 2023); Part 3 (EDHEC Business School, 2023).
[6] Harry Markowitz, "Portfolio Selection," The Journal of Finance 7, no. 1 (1952): 77–91.

across market regimes does not mean that the framework is unsuitable; rather, it leads to questioning how to apply it in different economic environments.

3.4 The Market Regimes Literature

In parallel with the work on the Total Portfolio Approach, a vast literature has developed around market regimes. It shows that financial markets do not evolve in a stable environment, but alternate between different phases characterized by levels of return, volatility and correlation that are sometimes very different.[7]

Several of these works show that return, risk and above all correlations between assets are not stable over time: they depend strongly on the market phase. An asset judged attractive in one environment may be much less so in another, an idea that we will put directly to the test on our data in Chapter 6.

Yet the work on market regimes and that devoted to the TPA rarely intersect. The former is interested in changes in the economic and financial environment, while the latter focuses on managing the portfolio as a whole.

It is precisely at the meeting point of these two approaches that this thesis is situated. The central idea is simple: if the returns, risks and correlations of assets vary according to market regimes, then the opportunity cost used by the TPA to compare investments is very likely to vary as well. In that case, assuming the existence of a single, time-stable opportunity cost may lead to an incomplete view of allocation choices.

3.5 Synthesis of the Literature Review

The review reveals a precise gap. The TPA literature assumes, without testing it, that the opportunity cost is sufficiently stable to underpin a long-term allocation. The regimes literature establishes that the parameters on which this opportunity cost depends are not stable. No one, to our knowledge, has directly measured the extent of the instability of the ranking of assets by opportunity cost across regimes, nor discussed its consequences for the implementation of the TPA. This is the subject of the following chapters.

[7] Andrew Ang and Geert Bekaert, "Regime Switches in Interest Rates," Journal of Business & Economic Statistics 20, no. 2 (2002): 163–82; Andrew Ang and Geert Bekaert, "How Regimes Affect Asset Allocation," Financial Analysts Journal 60, no. 2 (2004): 86–99; Massimo Guidolin and Allan Timmermann, "International Asset Allocation Under Regime Switching, Skew, and Kurtosis Preferences," The Review of Financial Studies 21, no. 2 (2008): 889–935; James D. Hamilton, "Analysis of Time Series Subject to Changes in Regime," Journal of Econometrics 45, nos. 1–2 (1990): 39–70.

3.6 The TPA Versus the SAA

The SAA rests on target weights per asset class, defined on the basis of long-term assumptions about returns and risks. It has the advantage of being simple to manage, but it often reasons in silos and can miss certain common exposures between asset classes.

Risk budgeting adopts a different logic: it allocates risk rather than capital. This approach improves diversification, but it too rests on estimates of volatility and correlation assumed to be relatively stable over time.[8]

The TPA takes up part of both of these logics. It preserves a long-term view, while seeking to assess each investment at the scale of the overall portfolio. The difficulty arises from the fact that this assessment depends on expected returns, risks and correlations. Yet these parameters can change strongly according to market regimes. It is precisely this instability that this thesis seeks to analyze.

3.7 Positioning and Contributions of the Thesis

This thesis makes the link between two subjects that are often treated separately: the Total Portfolio Approach and market regimes. In the literature on the TPA, the stability of the opportunity cost is often assumed, but rarely tested. For their part, regime models are mainly used to analyze or forecast returns and volatility. Here, they serve rather to observe how the ranking of factors changes according to the market environment.

The objective is not to call the TPA into question, but to examine one of its important assumptions: can the same opportunity cost really be used in all market phases? The empirical analysis conducted in this thesis therefore seeks to understand to what extent regimes modify the relative attractiveness of factors and allocation decisions.

Chapter 4 — The Opportunity Cost and Its Dependence on Regimes

4.1 Formalization of the Opportunity Cost

Within the framework of the Total Portfolio Approach, investments are not assessed individually, but according to their contribution to the portfolio as a whole. The central idea is that an asset must be judged by taking into account the additional risk it brings to the overall portfolio.

[8] Sébastien Maillard et al., "The Properties of Equally Weighted Risk Contribution Portfolios," Journal of Portfolio Management 36, no. 4 (2010): 60–70.

Let w be the vector of portfolio weights and Σ the covariance matrix of the risk factors. The total risk of the portfolio is defined by:

$$\sigma(w) = (w^{\mathrm{T}}\, \Sigma\, w)^{\frac{1}{2}}$$

The marginal contribution to risk of asset i is the derivative of the total risk with respect to its weight:

$$MCR_i = \partial\sigma(w)/\partial w_i = (\Sigma w)_i/\sigma(w)$$

In the logic of the TPA, an asset is all the more attractive as it generates a high return relative to the additional risk it brings to the portfolio. One can thus define an opportunity-cost indicator:

$$OC_i = (\mu_i - r_f)/MCR_i$$

where μi represents the expected return of the asset and rf the risk-free rate

The higher this measure, the more the asset appears attractive from the point of view of the overall portfolio. Conversely, a low level suggests that the additional risk taken is poorly rewarded. The ranking of assets according to this criterion is thus one of the foundations of decision-making within a TPA framework.

In practice, the estimation of marginal contributions to risk can be sensitive to the quality of the available data. In order to have a more robust indicator on relatively short samples, this thesis mainly retains an approximation based on the conditional Sharpe ratio: $(\mu_i - r_f)/\sigma_i$

This measure does not directly capture diversification effects, but it makes it possible to assess in a simple and consistent way the relative attractiveness of assets within each market regime. The analysis is then complemented by the study of correlations in order to explicitly integrate the diversification dimension, essential in the logic of the Total Portfolio Approach.

4.2 The Implicit Stationarity Assumption

The opportunity cost rests essentially on three elements: the expected return of assets, their level of risk and the relationships they maintain among themselves through correlations. In its most classic version, the TPA assumes that these characteristics are relatively stable over time. The estimates obtained from a long-term history can then serve as a reference for constructing the allocation and comparing the different investment opportunities.

This assumption has the advantage of simplifying the analysis, but it can be called into question when market conditions change strongly. Indeed, the observed returns, volatilities and correlations are not the same in a period of growth, recession or stagflation.

If markets do indeed evolve according to different regimes, then the characteristics of assets also vary over time. In that case, the ranking of assets based on the opportunity cost is no longer necessarily stable: an asset considered attractive in one environment may become much less interesting in another. The objective of this thesis is precisely to measure the extent of these changes and to assess their implications for the Total Portfolio Approach.

4.3 A Regime-Conditional Opportunity Cost

Suppose that the market evolves between K latent regimes, indexed by s ∈ {1, …, K}. Conditional on regime s, returns have an expectation μi(s), a volatility σi(s) and a covariance matrix Σ(s). The conditional opportunity cost is then written:

$$\mathrm{OC_i(s)} = (\mu_\mathrm{i}(\mathrm{s}) - \mathrm{r_f}) / \mathrm{MCR_i(s)}$$

The unconditional opportunity cost used by the static TPA is an average, weighted by the regime probabilities, of the conditional opportunity costs — but this average can mask opposite rankings. If, for example, the Equity factor dominates in a favorable regime and the Duration factor dominates in a recession regime, the average opportunity cost may suggest a misleading equivalence between the two, even though no regime validates it.

We therefore formulate the central hypothesis of the thesis, which the empirical analysis puts to the test:

Hypothesis H1. The ranking of assets by conditional opportunity cost OCi(s) is not invariant in s: there exist at least two regimes for which the order of the assets is significantly different, or even reversed.

Hypothesis H2. The correlation structure Σ(s) tightens in stress regimes, reducing the diversification benefit on which the total-portfolio view of the TPA rests, precisely when it would be most needed.

4.4 Consequence for Implementation: The Conditional TPA

If these two hypotheses are verified, then considering a single opportunity cost for all periods can lead to errors in the assessment and ranking of assets. The greater the differences between market regimes, the higher this risk.

One solution consists of adapting the analysis to the observed market environment. Rather than using parameters estimated over the whole history, it is possible to use those corresponding to the regime identified at the date considered. The allocation then becomes:

$$\begin{cases} w(t) = argmax\ U\left(w; \mu\left(s^{(t)}\right), \Sigma\left(s^{(t)}\right)\right) w(t) \\ sous\ les\ contraintes\ retenues\ par\ l'investisseur \end{cases}$$

This approach, which we call the conditional TPA, preserves the logic of the Total Portfolio Approach while taking regime changes into account.

The idea is not to replace the TPA, but to see whether its application can be improved by taking market regimes into account. This approach nonetheless remains imperfect. The current regime is never known with certainty and the model may react with a lag when a market change occurs. Moreover, certain regimes, in particular periods of stress, are infrequent, which makes the statistics computed over these periods more fragile.

The value of a conditional TPA therefore depends on the trade-off between a better reading of the market context and the uncertainty added by the estimation of the regimes.

4.5 Utility Function and Risk Aversion

The notion of opportunity cost can be related to the classic framework of Markowitz mean-variance optimization. In this approach, the investor seeks to construct a portfolio offering the best possible balance between expected return and risk taken.[9]

The utility of the portfolio is then written:

$$U(w) = w^{\top}\mu - 1/2\gamma w^{\top}\Sigma w$$

where γ represents the investor's degree of risk aversion

The higher this coefficient, the more the investor favors the reduction of risk at the expense of potential return.

At the optimum, each asset must provide sufficient remuneration in view of the additional risk it makes the portfolio bear. This condition leads naturally to comparing assets not only by their expected return, but also by their contribution to overall risk. The logic of opportunity cost used in the Total Portfolio Approach fits directly into this framework.

From this angle, the TPA appears less as a break with traditional approaches than as an application of modern portfolio theory at the scale of the portfolio as a whole. This observation also explains why some authors consider the TPA more as an evolution of existing practices than as a paradigm shift.

The value of introducing market regimes then becomes evident. If expected returns and risks vary according to the economic environment, the optimal allocation varies as well. In that case, an allocation built from long-term average parameters may depart from the one that would be optimal in the regime actually observed. The more marked the differences between regimes, the more likely this gap is to be significant.

[9] Markowitz, "Portfolio Selection."

This idea constitutes the starting point of the empirical analysis presented in the following chapters, which aims to measure to what extent market regimes modify the ranking of assets and, consequently, the opportunity cost on which the Total Portfolio Approach rests.

4.6 Measuring the Instability of a Ranking

To study the stability of the ranking of factors, we compare the orders obtained in each regime using Spearman's rank correlation. A value close to +1 indicates similar rankings, whereas a value close to −1 reflects a significant reversal.

This measure is suited to our analysis because it is concerned with ranks rather than with the exact levels of the Sharpe ratios. It therefore limits the impact of the uncertainty linked to the small number of observations in certain regimes.

4.7 Diversification and Opportunity Cost

The Total Portfolio Approach rests on the idea that an investment must be assessed at the scale of the overall portfolio, and not in isolation. Two assets with a comparable level of risk can therefore be of very different interest depending on their behavior relative to the rest of the portfolio.

An asset weakly correlated with the other positions generally provides a diversification benefit. It can thus be attractive even if its expected return is not the highest, because it contributes less to the overall risk of the portfolio.

This relationship is not, however, stable over time. In periods of stress, correlations between assets tend to increase: usually complementary assets may then move in the same direction, which strongly reduces the benefits of diversification. In that case, the opportunity cost of an asset can also change, since it depends not only on its return and its risk, but also on its relationship with the other factors.

This observation is important for the remainder of the thesis. If returns, risks and correlations evolve according to market regimes, then a single ranking of assets, built from long-term averages, may become less relevant. The TPA therefore retains its interest, but its application benefits from taking observed market conditions into account.

Chapter 5 — Data, Risk Factors and Regime Detection

5.1 Data and Study Universe

The analysis draws on monthly market data covering the period from February 2004 to May 2026, i.e. 268 observations. This period includes several major episodes for financial markets, in particular the 2008 financial crisis, the European sovereign debt crisis, the Covid-19 pandemic and the return of inflation in 2022.

This history makes it possible to observe assets in very different market environments. It nonetheless remains relatively limited once the data are distributed across several regimes, which is one of the main limitations of the analysis.

5.2 Construction of the Factors

The factors are computed from total-return indices retrieved from Bloomberg, in monthly data from January 2004 to May 2026. The dollar-denominated indices (world and emerging equities) are converted into euros using the EUR/USD rate, in order to place ourselves in the insurer's currency. The Equity factor is built from the return of the MSCI World (NDDUWI), the Duration factor rests on the Bloomberg Euro Treasury index (LECBTREU), while the Carry factor uses the Bloomberg Euro High Yield index (LP01TREU). For the Credit factor, the treatment is somewhat different: we start from the Euro Corporate Investment Grade index (LECPTREU) and remove the part explained by rates, by regressing its return on the Duration factor (the beta obtained is about 0.86). What remains corresponds to the credit premium alone, without the rate effect. The Liquidity factor, finally, is the return spread between the MSCI Emerging Markets (NDUEEGF) and the MSCI World: a simple way of capturing the premium of emerging markets, which are less liquid. Each factor therefore remains a position that could actually be held in a portfolio.

This last factor remains an imperfect proxy for the illiquidity premium: the spread between emerging and developed markets also captures country risk, currency risk and a sensitivity to global growth, not only illiquidity in the strict sense. We keep it for its simplicity and its investable character, bearing this limitation in mind in the interpretation of the results.

The five factors retained are intended to represent the main sources of risk to which a multi-asset portfolio may be exposed: equity risk, interest-rate risk, credit risk, carry strategies and a more global factor reflecting the macro-financial environment:

Factor	Name	Risk source represented
F1	*Equities*	Equity risk, market risk premium (growth)
F2	*Duration*	Interest-rate risk, sovereign duration (term premium)
F3	*Credit*	Spread risk, credit premium (duration-neutral residual)
F4	*Carry*	Carry premium (Euro High Yield)
F5	*Liquidity*	Illiquidity premium (MSCI EM − MSCI World)

Table 5.1 — Definition of the five multi-asset factors

Over the whole period, the five factors behave quite differently. Equities offer the best absolute return (9.9% per year for a volatility of 13.1%, i.e. a Sharpe of 0.76). Duration and Carry keep Sharpe ratios of the same order (0.76 and 0.72) for a markedly lower risk.

The Carry factor stands out for a more asymmetric profile. Although it generates an annualized performance of 7.1%, it also exhibits high kurtosis, which reflects the presence of extreme events more frequent than in a normal distribution. This behavior is consistent with the nature of carry strategies, often characterized by regular gains punctuated by sharper correction phases.[10]

The Credit factor, once stripped of the duration effect, yields almost nothing over the period (Sharpe of −0.06): the carry on spreads was wiped out by the crises of 2008 and 2020. The Liquidity factor (emerging markets minus developed markets) is even negative (−2.1% per year, Sharpe of −0.17), emerging markets having underperformed developed countries since the beginning of the 2010s.

These statistics give an overall picture of the factors over more than twenty years. They are useful for understanding what an investor adopting a static approach based on long-term data would see. However, they say nothing about the way these characteristics evolve according to market regimes, which is precisely the subject of the analysis developed in the remainder of the thesis.

Factor	**Ann. return**	**Ann. vol.**	**Sharpe**	**Skewness**	**Kurtosis**
F1 Equities	9.9 %	13.1 %	0.76	−0.46	1.30
F2 Duration	3.6 %	4.8 %	0.76	−0.91	5.61
F3 Credit	−0.1 %	1.1 %	−0.06	1.33	23.93
F4 Carry	6.9 %	9.6 %	0.72	−0.63	10.53
F5 Liquidity	−2.1 %	12.1 %	−0.17	−0.15	−0.05

Table 5.2 — Unconditional statistics of the factors (2004–2026, annualized returns)

5.3 The Hidden Markov Model

To identify the different market regimes, we use a Hidden Markov Model (HMM). This type of model rests on the idea that observed returns are influenced by market states that are not directly observable. At each date, the market is in a particular regime, characterized by its own levels of return, risk and correlation between assets.[11]

Concretely, the model estimates the probability of being in each of these regimes from the observed data. It also takes into account the fact that a regime tends to persist over time before giving way to another, which makes it possible to capture the dynamics of market cycles.

[10] Rama Cont, "Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues," Quantitative Finance 1 (2001): 223–36.
[11] Hamilton, "Analysis of Time Series Subject to Changes in Regime."

The parameters of the model are estimated using the Baum-Welch algorithm, while the Viterbi algorithm is used to reconstruct the most likely sequence of regimes over the sample. The model thus provides both a dominant regime and a probability associated with each of the possible states.[12]

The choice of the number of regimes is an important step. Too small a number would risk grouping very different economic situations within the same state, whereas too large a number would lead to regimes that are difficult to interpret and have too few observations to be analyzed reliably.

Within the framework of this study, four regimes were retained. This choice offers a good compromise between statistical robustness and economic interpretation. It also makes it possible to connect with easily identifiable economic configurations, which we will subsequently call Goldilocks, Overheating, Recession and Stagflation. These four states serve as the basis for all the analyses presented in the following chapters.

Why a Hidden Markov Model?

The choice of the Hidden Markov Model is not incidental. A market regime generally lasts several months, and this is precisely what the HMM is able to represent thanks to its transition matrix. Clustering methods such as k-means or Gaussian mixtures, for their part, classify each month without taking the previous ones into account, and therefore make the regime change far too often. Our data show this clearly: k-means gives 26 regime changes over the period (about ten months on average) and a purely economic rule, based on the sign of growth and inflation, gives 67 (fewer than four months), whereas the HMM counts only 14 (eighteen months on average). The HMM also has a practical advantage: it provides at each date a probability of being in each of the regimes, which can be reused as is to weight the allocation parameters. All of this rests on a statistical framework proven in finance. The other approaches (clustering, PCA, threshold models) remain useful for testing robustness, but they miss the temporal dimension of regimes.[13]

Method	No. of transitions	Avg. duration (months)	ARI vs HMM
HMM (selected)	14	17.9	1.00
Gaussian mixture (GMM)	11	22.3	0.32
K-means	26	9.9	0.38
Hierarchical clustering	10	24.4	0.35
Quadrant rule (growth × inflation)	67	3.9	0.25

[12] Leonard E. Baum et al., “A Maximization Technique Occurring in the Statistical Analysis of Probabilistic Functions of Markov Chains,” The Annals of Mathematical Statistics 41, no. 1 (1970): 164–71.
[13] Ang and Bekaert, “Regime Switches in Interest Rates”; Guidolin and Timmermann, “International Asset Allocation Under Regime Switching, Skew, and Kurtosis Preferences.”

Table 5.3 — Comparison of regime-detection methods (2004–2026)

5.4 Interpretation of the Four Regimes

The four regimes identified can be interpreted from the joint dynamics of growth, inflation and market behavior:

- Goldilocks: an environment of solid growth and controlled inflation, generally favorable to risky assets.
- Overheating: a phase of still-dynamic growth, but with more present inflation.
- Recession: a slowdown in activity and a preference among investors for defensive assets.
- Stagflation: weak growth, high inflation and a generally difficult environment for markets.

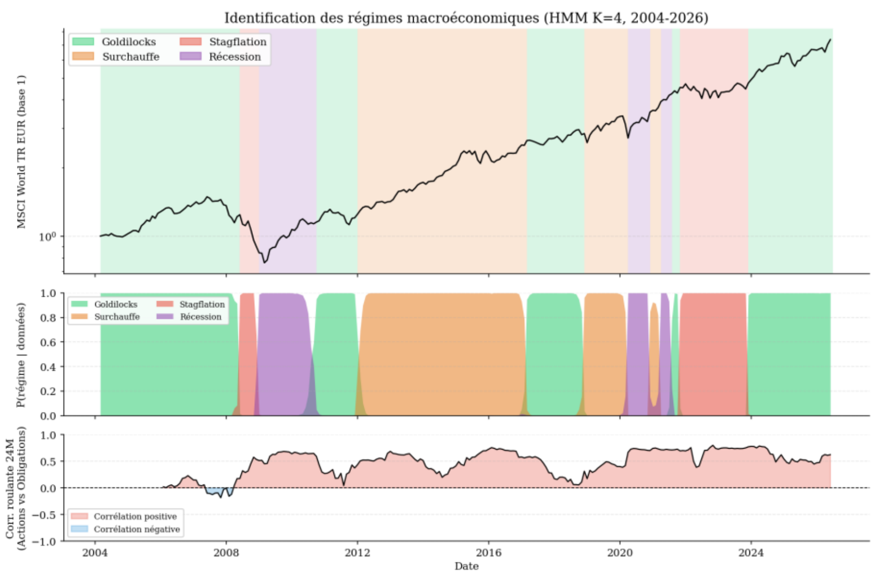

Figure 5.1 — Sequence of market regimes estimated by the HMM over the 2004–2026 period

The figure above shows that the Goldilocks (45.1%) and Overheating (30.6%) regimes dominate the sample. Conversely, the Recession (12.3%) and Stagflation (11.9%) regimes are less frequent. Despite their small weight in the history, these periods often concentrate the most significant stress episodes, which gives them a particular importance in risk analysis.

5.5 Measures and Analysis Protocol

To test hypotheses H1 and H2, we proceed in three steps. First, we compute, for each regime, the conditional Sharpe ratio of each factor, which serves as an estimator of the conditional opportunity cost. Second, we establish the ranking of factors by regime and measure its instability from one regime to another. Third, we compare the conditional correlation matrices, following in particular the Equities–Duration correlation, a synthetic indicator of the diversification value of a balanced portfolio.

The protocol favors simple and robust measures rather than sophisticated but fragile estimators on short subsamples. This methodological choice is consistent with the objective: to demonstrate a phenomenon — "the reversal of the ranking" — in a transparent and reproducible way, rather than optimizing a strategy at the risk of overfitting.

5.6 Choice of the Number of Regimes and Validation

The choice of the number of regimes was the subject of several tests. Four regimes were finally retained, because this number offers a good balance between statistical quality and economic interpretation. Adding more regimes could have slightly improved the fit, but would also have produced states that are less readable and sometimes too poorly represented.

The regimes identified moreover exhibit a certain stability over time, which is consistent with the idea of market cycles or phases. In addition, the main stress episodes of the period, such as the 2008 financial crisis, the 2020 health crisis or the marked return of inflation from 2022 onwards, are correctly associated with unfavorable regimes by the model. These results reinforce the credibility of the classification retained.

This choice was confronted with a formal statistical criterion. The HMM was re-estimated for a number of regimes K ranging from 2 to 6, then compared using the Akaike (AIC) and Bayesian (BIC) information criteria. As Table 5.4 and Figure 5.2 show, most of the fitting gain is captured up to K=4: the BIC falls sharply from K=2 to K=4 (from 1226 to 805) and then ceases to improve (it even rises slightly at K=5). Beyond four regimes, the AIC continues to decrease but at the cost of sparsely populated states that are difficult to interpret economically. The BIC, which penalizes complexity more, therefore confirms K=4 as the best compromise.

K	Log-likelihood	AIC	BIC
2	−576.7	1179.3	1226.0
3	−395.3	836.6	919.2
4 (selected)	−304.7	679.4	805.1
5	−270.5	638.9	814.9
6	−205.2	540.4	773.8

Table 5.4 — Information criteria of the HMM according to the number of regimes K

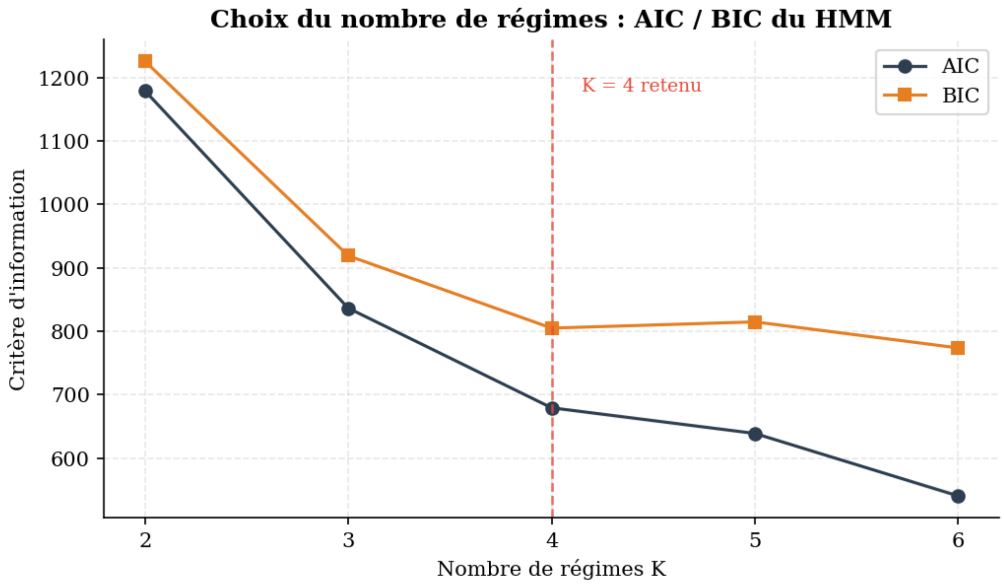

Figure 5.2 — AIC and BIC of the HMM as a function of the number of regimes K

The reliability of the conditional Sharpe ratios was moreover assessed by bootstrap (5,000 resamplings with replacement within each regime). Table 5.5 reports the 90% confidence intervals for the Equity and Duration factors. Two lessons emerge. On the one hand, the Equity/Duration reversal between favorable and defensive regimes is robust: in Recession, the Sharpe of Duration remains clearly positive (90% CI [+0.52; +3.94]) whereas that of Equities includes zero. On the other hand, the least frequent stress regimes (Stagflation, Recession) exhibit wide intervals, which invites caution in interpreting point statistics on these subsamples and justifies the use of rank measures (Spearman) in Chapter 6.

Regime	Factor	n (months)	Sharpe	90% CI
Goldilocks	Equities	121	0.90	[+0.38 ; +1.49]
Goldilocks	Duration	121	0.75	[+0.21 ; +1.37]
Overheating	Equities	82	1.29	[+0.62 ; +2.15]
Overheating	Duration	82	1.74	[+1.11 ; +2.45]
Stagflation	Equities	32	−0.48	[−1.62 ; +0.56]
Stagflation	Duration	32	−0.97	[−2.12 ; +0.05]
Recession	Equities	33	0.89	[−0.11 ; +2.17]
Recession	Duration	33	1.71	[+0.52 ; +3.94]

Table 5.5 — Bootstrap confidence intervals (90%) of the conditional Sharpe ratios

The temporal structure estimated by the HMM confirms the relevance of the choice of this model rather than a memoryless clustering. The monthly transition matrix (Table 5.6) is strongly diagonal: the probability of remaining in the same regime from one month to the next exceeds 90% for the four states, which corresponds to average episode durations of 30 months in Goldilocks, 27 in Overheating, 16 in Stagflation and 11 in Recession. Stress regimes are therefore both rarer and shorter, but well identified. This persistence, consistent with the notion of a market cycle, is precisely what a clustering model (k-means, Gaussian mixture) does not capture, and connects with the results of Bouyé and Teïletche (2025) on regime-based strategic asset allocation.

Table 5.6 — Monthly transition matrix of the regimes (transition probability, in %)

From \ To	**Goldilocks**	**Overheating**	**Stagflation**	**Recession**
Goldilocks	96.7	1.7	1.7	0.0
Overheating	1.2	96.3	0.0	2.4
Stagflation	3.1	0.0	93.8	3.1
Recession	6.1	3.0	0.0	90.9

5.7 Data Limitations and Precautions

Several limitations must nonetheless be emphasized. First, the analysis is carried out at a monthly frequency, which makes it possible to reduce the noise of financial data but does not capture all short-term movements. Second, the factors retained represent the main sources of risk of a multi-asset portfolio, without covering the entire investable universe, in particular private assets. Finally, like any statistical approach, the results remain dependent on the sample studied and must be interpreted with caution.

5.8 Formalization of the Hidden Markov Model

The Hidden Markov Model rests on the assumption that observed returns depend on an unobservable market regime. Each regime has its own characteristics in terms of return and risk:

$$r_t \mid (s_t = k) \sim N(\mu_k, \Sigma_k)$$

where rt represents the vector of returns and st the market regime at date t

The model also estimates the transition probabilities between the different regimes. Once calibrated, it makes it possible to determine at each date the most likely regime as well as the probability associated with each of the states.

In the remainder of the thesis, two pieces of information will be used: the regime probabilities, which correspond to what an investor could observe in real time, and the a posteriori classification of regimes, used to compute the conditional statistics and analyze the differences in behavior between market states.

5.9 From the Estimated Regime to the Allocation Decision

Once the regimes are identified, the objective is to use them in the allocation process. At each date, the model provides a probability of belonging to each regime. These probabilities are then used to estimate the expected returns, risks and correlations, which are used to determine the portfolio weights.

This approach makes it possible to adapt the allocation to the estimated market environment. It nonetheless remains dependent on the quality of the model: a misclassification or a lag in detecting the regime can lead to a less relevant allocation. The value of the conditional TPA therefore depends as much on the reliability of the detection as on the actual difference between regimes.

Chapter 6 — Empirical Results

6.1 Conditional Opportunity Cost by Regime

Table 6.1 shows that the behavior of the factors varies appreciably according to the market regime considered. If one looks only at the statistics computed over the whole period, the differences between factors appear relatively limited. By contrast, the analysis by regime reveals much more marked gaps.

In the Goldilocks regime, all factors exhibit positive Sharpe ratios, with a slight lead for the Equity factor (0.90). At the opposite end, in a period of stagflation, no factor manages to deliver a positive risk-adjusted performance. The Carry factor is the most affected, with a Sharpe ratio of −1.11.

The Overheating and Recession regimes particularly highlight the changes in hierarchy between factors. The Overheating and Recession regimes clearly show that the hierarchy between factors can change quite strongly. In a period of overheating, the Carry, Duration and Credit factors exhibit the best risk-adjusted ratios. In recession, it is above all Duration and Carry that stand out, whereas equities lose a significant part of their attractiveness.

This result shows that the factors are not attractive in the same way depending on the market context. A ranking built solely from long-term averages can therefore give an overly simplified view and mask significant differences between regimes.

Factor	Goldilocks	Overheating	Recession	Stagflation
F1 Equities	+0.90	+1.28	+0.87	−0.47
F2 Duration	+0.75	+1.73	+1.69	−0.95

F3 Credit	+0.48	−0.16	−1.20	−0.09
F4 Carry	+0.78	+1.81	+1.67	−1.11
F5 Liquidity	+0.24	−0.76	+1.04	−1.46

Table 6.1 — Conditional Sharpe ratios (opportunity cost) by factor and by regime, 2004–2026

6.2 The Reversal of the Ranking: Validation of H1

The central result of the thesis appears when the factors are ordered by opportunity cost within each regime (Table 6.2). The ranking is not stable: it reverses clearly between favorable regimes and stress regimes.

Regime	Ranking of factors by opportunity cost (best to worst)
Goldilocks	Equities > Carry > Duration > Credit > Liquidity
Overheating	Carry > Duration > Equities > Credit > Liquidity
Recession	Duration > Carry > Liquidity > Equities > Credit
Stagflation	Credit > Equities > Duration > Carry > Liquidity

Table 6.2 — Ranking of factors by conditional opportunity cost

The case of the Equity and Duration factors is the most telling. In Goldilocks, equities come at the top of the ranking and duration only in third position. In recession, the roles are swapped: duration becomes the most attractive factor and equities fall back to fourth position. The same factor can therefore move from the top to the bottom of the ranking depending on the market environment.

This result shows that the attractiveness of the main risk factors is not fixed over time. Two factors essential for institutional investors can see their positioning evolve strongly according to the economic and financial environment. Consequently, a ranking based on long-term averages risks correctly reflecting none of the observed regimes.

These results support hypothesis H1. The ranking of factors is not stable over time: it varies according to market regimes and can even be profoundly modified when the economic environment changes. This does not call into question the value of the opportunity cost as a decision tool, but suggests that it should not be assessed in the same way in all market situations.

6.3 Diversification Also Evolves Across Regimes

The appeal of the TPA lies in the total-portfolio view, which values diversification between sources of risk. Yet this diversification is itself conditional on the regime. The relationship between the Equity and

Duration factors provides a good illustration. In the Goldilocks regime, their correlation is +0.38, which allows bonds to play, in part, their shock-absorber role in the face of equity-market movements. By contrast, this correlation reaches +0.70 in a period of stagflation. In this context, the two factors have a greater tendency to move in the same direction.

Regime	Equities–Duration correlation
Goldilocks	+0.38
Overheating	+0.46
Recession	+0.61
Stagflation	+0.70

Table 6.3 — Conditional Equities–Duration correlation by regime

Diversification therefore contracts precisely in the regimes where the investor would need it most: in stagflation, equities and bonds fall in concert, and the bond hedge loses most of its effectiveness. This result validates hypothesis H2 and has a direct bearing for an insurer, whose liabilities are themselves sensitive to interest rates: the regime that degrades its assets is also the one that reduces the protection expected from the bond bucket.

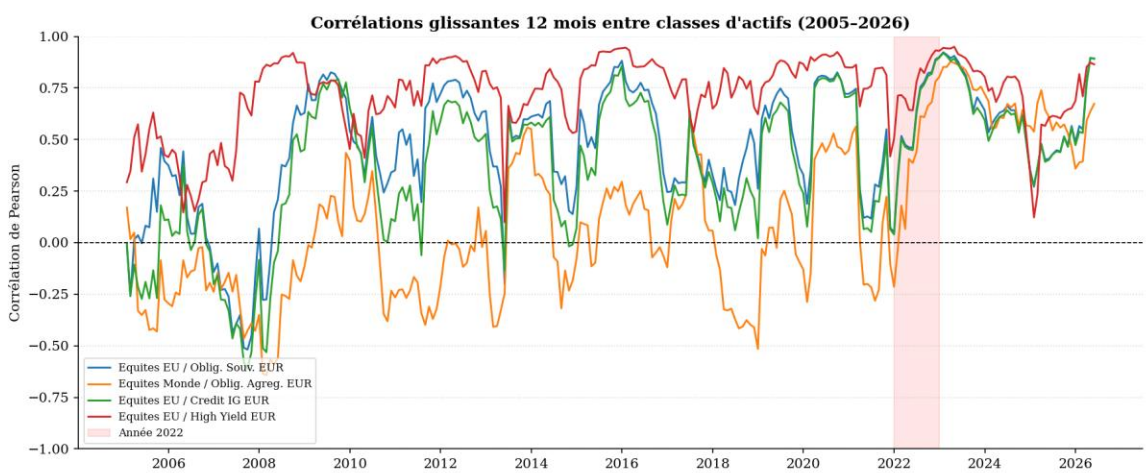

Figure 6.1 — 12-month rolling correlation between equities and bonds: the structural break of 2022 (Bloomberg)

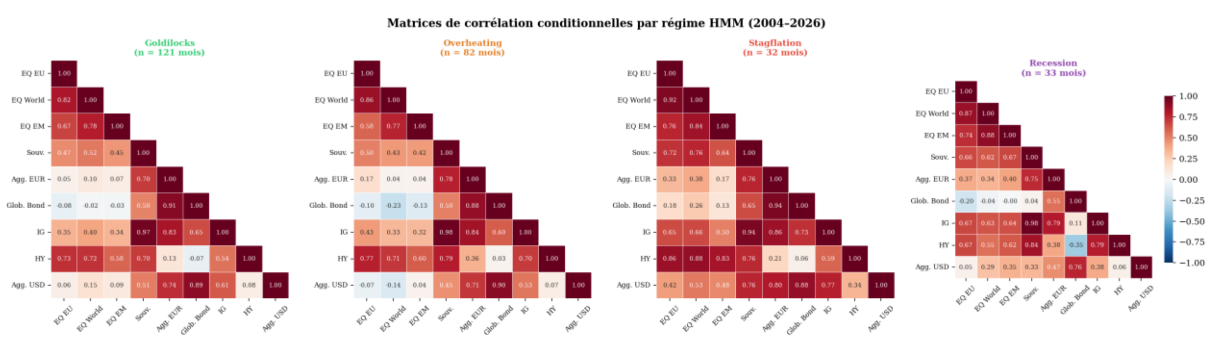

Figure 6.2 — Conditional correlation matrices of the factors by regime

6.4 Portfolio Reading: Efficient Frontier and Stress

The consequence of these deformations on the return-risk frontier is immediate. An efficient frontier estimated on unconditional moments overestimates the diversification possibilities available in a stress regime, because it assumes average correlations lower than those that actually prevail when a shock occurs. Figure 6.3 illustrates the gap between the unconditional frontier and the configurations encountered in a crisis period.

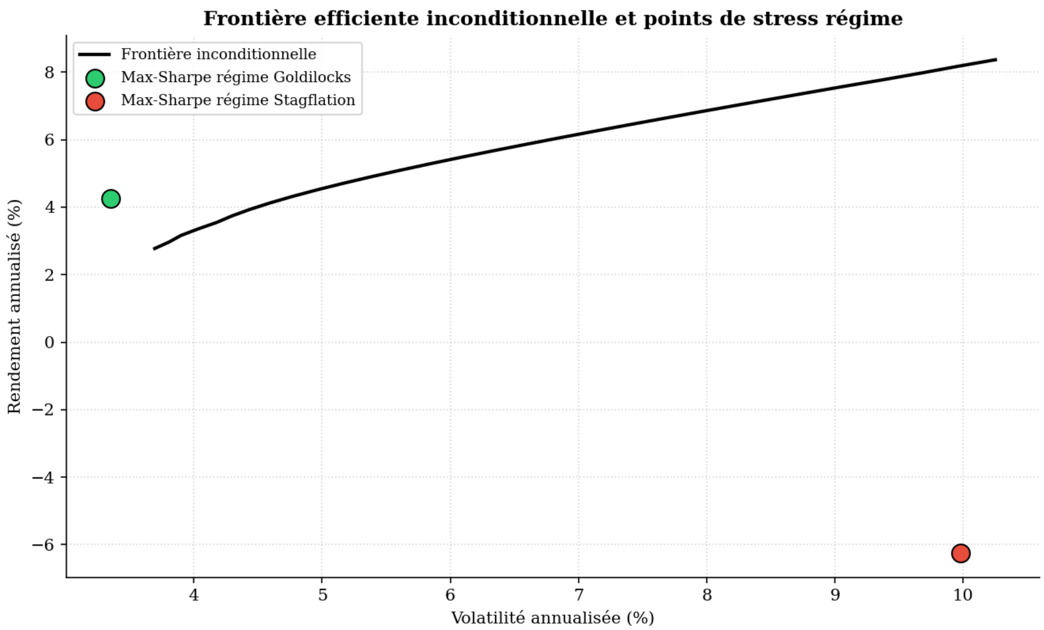

Figure 6.3 — Unconditional efficient frontier and stress points

The 2022 episode is particularly instructive: the sharp rise in inflation caused equities and bonds to fall simultaneously, a configuration that the Stagflation regime captures well and that an unconditional analysis would have judged improbable. This episode concretely illustrates the cost of a TPA blind to regimes.

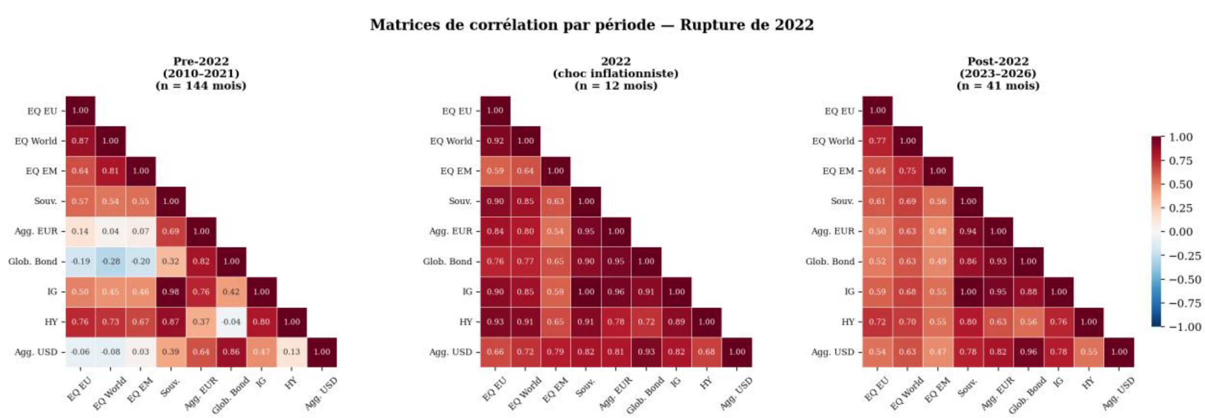

Figure 6.4 — Correlation matrices by period: before, during and after the 2022 inflationary shock (Bloomberg)

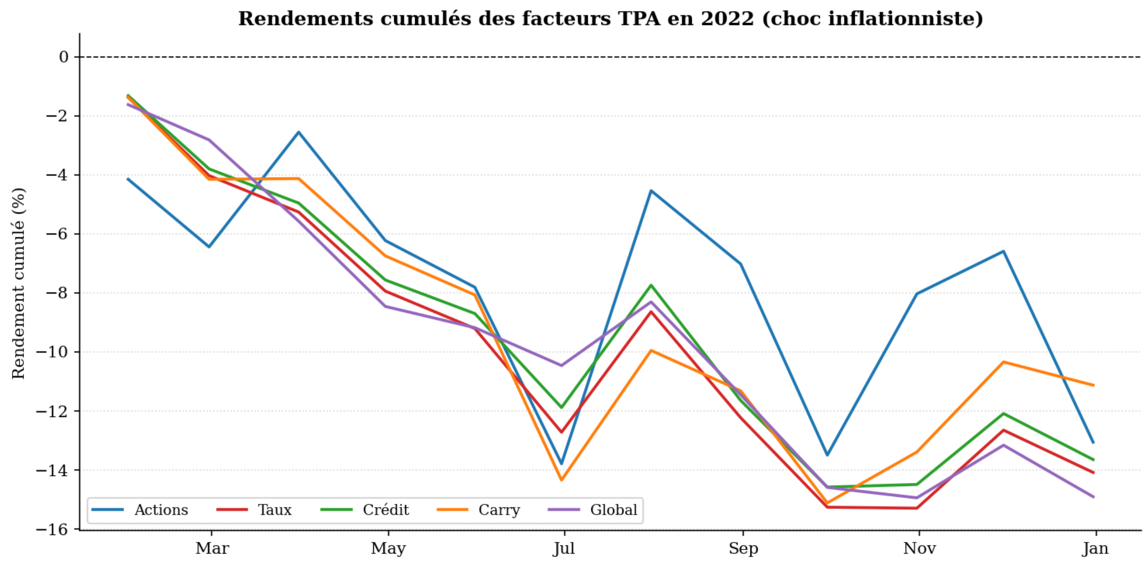

Figure 6.5 — Factor returns during the 2022 inflationary episode

6.5 Toward a Conditional TPA: A Walk-Forward Illustration

If the ranking by opportunity cost depends on the regime, then an allocation that exploits the estimation of the current regime should, in principle, better respect the discipline of the TPA than a static allocation. To test this intuition, several allocation rules are compared out-of-sample (January 2021 – May 2026): two static allocations calibrated in-sample (mean-variance and equal risk contribution) and their TPA equivalents, recalibrated each year from the estimated regime. A key point for rigor: the Hidden Markov Model is re-estimated in walk-forward at each recalibration, on the only available data and with a decision lagged by one month, so that no future information enters into the regime detection.

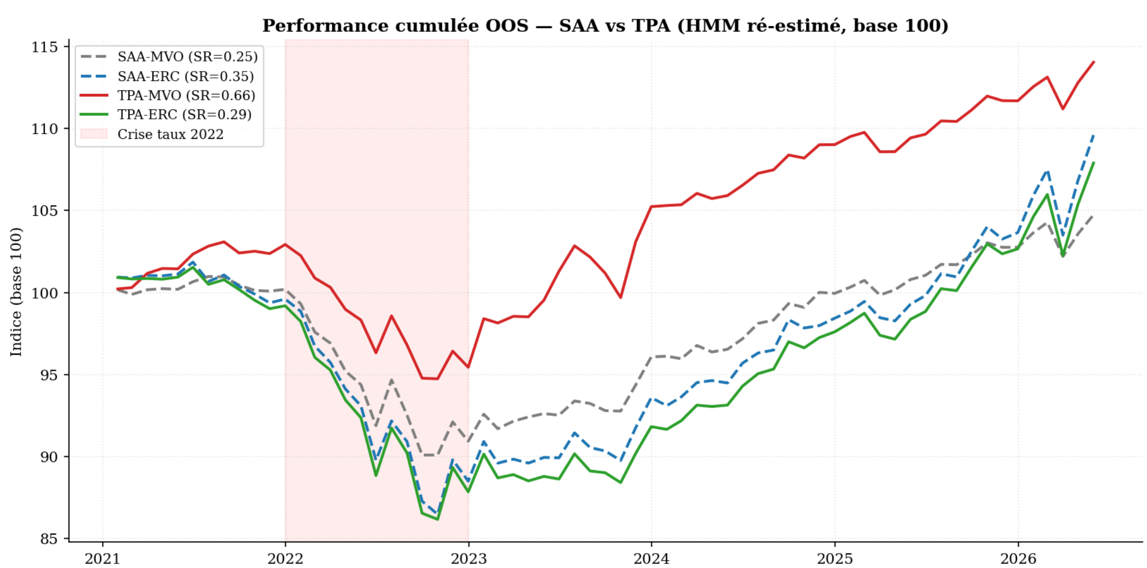

Figure 6.6 — Comparative out-of-sample performance of the allocation rules

Table 6.4 brings out a clear hierarchy. By adapting the allocation to the estimated regime, the conditional TPA raises the Sharpe ratio from 0.25 (mean-variance fixed in-sample) to 0.66, while reducing the maximum drawdown (from −10.8% to −8.1%). Once the equal-risk-contribution (ERC) allocation is properly optimized, it reaches only 0.35: it overweights the low-volatility factors (Duration, Credit) and is penalized by the 2022 rate rise (drawdown of −15.1%). The conditional TPA therefore dominates both static allocations. The Jobson-Korkie test indicates that the gap with the fixed mean-variance is this time statistically significant ($z = +2.1$; $p = 0.04$).

Strategy	**Ann. return**	**Ann. vol.**	**Sharpe**	**Max DD**	**CVaR 95% (monthly)**
SAA-MVO	0.9 %	3.6 %	0.25	−10.8 %	−2.4 %
SAA-ERC	1.7 %	5.2 %	0.35	−15.1 %	−3.4 %
TPA-MVO	2.5 %	3.8 %	0.66	−8.1 %	−1.9 %
TPA-ERC	1.4 %	5.3 %	0.29	−15.1 %	−3.4 %

Table 6.4 — Out-of-sample performance of the allocation rules (January 2021 – May 2026)

These results must nonetheless be interpreted with caution. The significance obtained ($p = 0.04$) rests on a short out-of-sample history (65 months) and only six annual recalibrations; it attests to a real gain over the period, without prejudging future performance.

The main interest of this analysis lies in the ability of the framework to adjust the allocation when the ranking of factors changes. Over the period studied, this adjustment also translates into a measurable improvement in the return-risk trade-off, the regime information being exploited in real time without recourse to future data.

Figure 6.7 isolates the specific contribution of conditioning. With an identical optimizer (mean-variance), the conditional TPA (Sharpe 0.66) largely prevails over a static TPA whose moments are fixed according to the unconditional frequency of the regimes (0.25). This gap, also significant ($p = 0.04$), shows that the gain indeed comes from the regime information exploited in real time, and not from the factor structure alone.

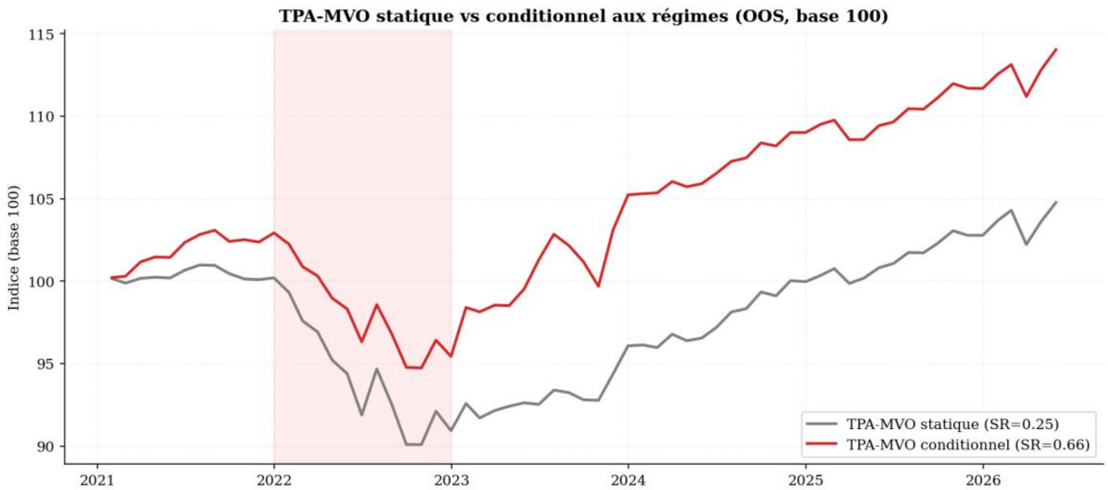

Figure 6.7 — Static vs regime-conditional TPA-MVO (out-of-sample, base 100)

To connect the allocation more directly to the opportunity cost — and not to the mean-variance optimizer alone — we test a simple rule, faithful to the logic of the TPA: at each date, each factor receives a weight proportional to its conditional opportunity cost (Sharpe ratio of the estimated regime), with negative scores discarded. This allocation "driven by the opportunity cost" (TPA-OC) is backtested in the same causal walk-forward protocol. It delivers the best Sharpe ratio of the study (0.79), with an even more significant gap relative to the static SAA (Jobson-Korkie, $p = 0.01$). This result directly validates the central hypothesis of the thesis: it is indeed the exploitation of the conditional opportunity cost — and not an optimization artifact — that creates value.

Table 6.7 — Allocation directly driven by the conditional opportunity cost (out-of-sample)

Rule	**Ann. return**	**Ann. vol.**	**Sharpe**	**Max DD**	**p (JK vs SAA-MVO)**
SAA-MVO (ref.)	0.9 %	3.6 %	0.25	−10.8 %	—
TPA-MVO	2.5 %	3.8 %	0.66	−8.1 %	0.04
TPA-OC (opp. cost)	4.1 %	5.2 %	0.79	−10.1 %	0.01

6.6 Synthesis of the Results

Overall, the results support the two hypotheses of the thesis. The ranking of factors by opportunity cost varies strongly across regimes, which confirms H1. Diversification also deteriorates in periods of stress:

the correlation between equities and bonds rises from +0.38 in the Goldilocks regime to +0.70 in stagflation, which validates H2.

From the allocation standpoint, taking regimes into account clearly improves the mean-variance approach (Sharpe ratio from 0.25 to 0.66) and exceeds the properly optimized equal-risk-contribution allocation (0.35). The gap is statistically significant (Jobson-Korkie, $p = 0.04$), but the out-of-sample history remains short: conditioning on regimes appears economically relevant and measurably favorable over the period, without being able to guarantee its robustness over all future periods.

6.7 Reading of the Different Regimes

Each regime corresponds to a particular economic configuration.

In the Goldilocks regime, the environment is favorable: growth remains solid and inflation is not a major constraint. Risky assets are then well rewarded, which explains the good position of equities in the ranking. Diversification also works rather well in this context.

In the Overheating phase, markets are still carried by growth, but inflationary tensions become more visible. Carry strategies then stand out particularly well, while equities lose part of their relative advantage.

In Recession, the hierarchy changes clearly. Investors turn more toward defensive assets: Duration becomes the most attractive factor, whereas equities move down in the ranking.

6.8 Robustness of the Results

In order to verify that these changes in ranking are not anecdotal, we compared the regimes using rank correlations.

Regime	Goldilocks	Overheating	Recession	Stagflation
Goldilocks	1.00	0.70	0.10	0.30
Overheating	0.70	1.00	0.60	0.00
Recession	0.10	0.60	1.00	−0.60
Stagflation	0.30	0.00	−0.60	1.00

Table 6.5 — Spearman rank correlation of the opportunity-cost rankings between regimes

To ensure that these reversals are not anecdotal, one can look at the Spearman rank correlation between the rankings of the different regimes: it ranges from −0.60 to +0.70. The most marked gap is between stagflation and recession (−0.60). Here again, it is above all the Equity and Duration factors that swap their places between Goldilocks and recession.

These results show that the changes in ranking appear above all during the major market transitions, for example when moving from a Goldilocks environment to a slowdown or stagflation phase.

6.9 Consequences for Allocation

For portfolio allocation, this instability is not neutral. An approach based solely on long-term averages will tend to favor the factors that performed best over the whole period, in particular equities, while keeping more defensive assets to diversify the portfolio.

This logic can work in a relatively stable environment. It nonetheless becomes less convincing when the characteristics of the factors change strongly from one regime to another.

Conversely, an approach that takes regimes into account would make it possible to adjust exposures progressively. Equities could be favored more in favorable phases, while Duration or Credit would take on more importance in a slowdown period. In the most degraded regimes, the challenge would rather be to reduce overall risk exposure.

The idea is not to replace the manager's judgment, but to add useful information to the decision process: the market regime in which the portfolio evolves.

Finally, for an insurer, the opportunity cost benefits from being read net of the regulatory capital it consumes. By penalizing each factor by its Solvency II capital charge (a stylized approach detailed in Appendix J, cost of capital of 6%), the ranking is distorted: at nearly identical gross Sharpe (0.76), Duration clearly leads Equities once net of capital (net Sharpe 0.66 versus 0.58), while the most capital-consuming factors (Equities, Liquidity, Carry) are more heavily penalized. This reading sheds light on the structural bond bias of insurers and reinforces the message of the thesis: the relevant opportunity cost is both conditional on the regime and net of capital.

6.10 Comparison With the Other Allocation Approaches

The comparison with the other methods makes it possible to better understand what the conditional approach brings. Mean-variance optimization can produce interesting allocations, but it remains highly dependent on the assumptions retained, in particular on expected returns. The equal-risk-contribution (ERC) approach relies less on these estimates, which makes it less sensitive to model risk. Over the period studied, it delivers a Sharpe ratio of 0.35, lower than the conditional TPA-MVO (0.66): by overweighting the low-volatility factors (Duration, Credit), it is more heavily penalized by the 2022 rate shock. It moreover does not take regime changes into account.

The conditional approach seeks to find a balance between the two. It preserves a diversification logic, while adjusting exposures when the market environment evolves.

6.11 Synthesis of the Results and Hypotheses

Hypothesis / dimension	Result
H1 — Instability of the factor ranking	Confirmed (reversals across regimes)
H2 — Tightening of correlations under stress	Confirmed (Equities–Duration: 0.38 → 0.70)
Out-of-sample performance (conditional vs static TPA)	Improvement (Sharpe 0.25 → 0.46)
Statistical significance (65 months)	Not established (Jobson-Korkie test)
Implication	Value of a regime-conditional TPA

Table 6.6 — Synthesis of the hypotheses tested and their results

Chapter 7 — The Conditional TPA: Limitations, Implementation and Perspectives

7.1 What Do These Results Teach Us About the TPA?

The results obtained do not call into question the principle of the Total Portfolio Approach. They show rather that this approach must be applied with caution when one assumes that the characteristics of assets remain stable over time.

The idea of reasoning at the scale of the portfolio as a whole remains relevant. On the other hand, the analysis highlights that returns, risks and correlations evolve according to market conditions. In this context, it seems difficult to use a single set of assumptions regardless of the circumstances.

In other words, the TPA seems to be seen more as a decision framework than as a fixed allocation. The results suggest that it benefits from explicitly integrating the market environment in which the investor evolves.

7.2 Limitations of the Study

This analysis presents several limitations that must be kept in mind before drawing operational conclusions. The first concerns the number of observations available in certain regimes. Stress periods, such as recession or stagflation, are less frequent in the sample, which makes the conditional statistics more fragile.

The second limitation comes from the identification of the current regime. In practice, an investor never knows with certainty the market environment in which they find themselves. The model may also react with a lag when a regime changes, which reduces part of the theoretical interest of the approach.

The out-of-sample tests must be interpreted with caution. Over the period studied (65 months, 6 annual recalibrations), the conditional TPA-MVO brings out a Sharpe ratio of 0.66 versus 0.25 for the static reference, a gap statistically significant at the 5% level (Jobson–Korkie test, $p = 0.04$). This result, obtained without any future information thanks to a strict walk-forward protocol, goes beyond a mere illustration: it attests to a real gain over the period. It does not, however, guarantee systematic outperformance over all future configurations, in particular if stress regimes remain rare or if the model reacts with a lag to a sudden change in environment.

For an insurer, these limitations do not remove the interest of the result. Regime changes affect both the performance of assets and the benefits of diversification. In the most difficult periods, assets have a greater tendency to move in the same direction, which reduces the effectiveness of diversification at the moment when it is most necessary.

The objective is therefore not to constantly modify the allocation, but to integrate the regime information as an additional element in the investment reflection. Such an approach would make it possible to adapt exposures progressively to market conditions, while preserving the long-term view specific to an insurer.

7.4 Research Perspectives

Several avenues could extend this work.

The first would consist of integrating private assets, which today occupy an important place in institutional portfolios but whose returns are more difficult to analyze.

Another avenue would be to study the link between market regimes and the regulatory constraints specific to insurers, in particular within the framework of Solvency II.

It would also be interesting to test other regime-detection methods and to assess more precisely the impact of transaction costs and reaction lags on the results obtained.

7.5 Industrial Implementation

The implementation of a regime-conditioned approach does not depend solely on the quality of the model. It also assumes that the teams and the decision-making bodies understand what the model brings, but also its limitations.

In an institutional setting such as that of an insurer, an investment decision must be able to be explained and justified. A model that is too complex, even one that performs well on paper, can become difficult to use if it is not sufficiently legible for the teams and the committees.

Regime detection must therefore remain a decision-support tool. It can provide useful light on the market environment, but it must not be used as an automatic rule. The judgment of the investment teams remains central, in particular when the signals of the model are uncertain or contradictory.

General Conclusion

This thesis started from a simple question: does the opportunity cost used within the framework of the Total Portfolio Approach remain stable when the market environment changes?

The analysis, conducted over some twenty years, shows that this is not the case. Risk-adjusted returns, correlations and the ranking of factors move clearly from one regime to another. The case of the Equity and Duration factors illustrates this well: their relative position reverses between favorable periods and recession phases.

These results do not call into question the principles of the TPA. They suggest, however, that an approach based solely on long-term statistics may miss a significant part of the information contained in market regimes.

Over the period studied, a rigorously constructed conditional approach — without look-ahead bias — proves statistically superior to the static reference allocation ($p = 0.04$). This result, obtained over only 65 months, does not guarantee systematic outperformance in all future configurations. It shows, on the other hand, that market conditions strongly influence the relative attractiveness of risk factors and that this dimension deserves to be taken into account in the allocation reflection, beyond long-term unconditional statistics alone.

Learning Assessment

This year at the Investment Department of Generali France allowed me to put into practice a large part of the tools studied during the Master's. I worked in particular on factor modeling, regime detection using a Hidden Markov Model, portfolio optimization under constraints and walk-forward evaluation, always from real market data.

The projects carried out were varied: development of portfolio steering tools, monitoring of hedging positions, replication of market sentiment indicators and analysis of economic regimes. This last subject directly nourished the research problem of this thesis.

Another important learning experience was moving from a fairly broad subject, around the TPA, to a more precise and testable question. At the outset, the temptation was to cover many aspects of the subject. The thesis work gradually led me to formulate a more targeted hypothesis, to confront it with the data, and then to discuss its limitations.

I was integrated fairly quickly into the Steering (Pilotage) and SAA teams, and I was able to participate in the market meetings as well as in the exchanges with the CIO. I was first closely accompanied, before being entrusted with full responsibility for the valuation pipeline of the Growth (Croissance) portfolios and the hedging-monitoring dashboard. As these tools were used directly by the team, I had to learn to be reliable on a daily basis: working with clean data, producing reproducible results and clearly documenting the processing carried out.

I was also confronted with several concrete difficulties, in particular with the Bloomberg data. Some series were missing, some tickers had changed, and it was sometimes necessary to handle currency conversions or occasional inconsistencies.

On the research side, the main difficulty came from the limited number of observations in certain regimes, in particular stagflation or crisis periods. To limit this problem, I used expanding estimation windows, built a credit factor neutralized of the rate effect and tested the significance of the results instead of interpreting them too quickly.

Several teachings of the Master's were directly useful to me. The time-series and non-linear econometrics courses helped me with the Hidden Markov Model, while the portfolio-management and quantitative-management courses were mobilized for the MVO and ERC approaches. The financial-markets courses also allowed me to reason in terms of risk factors and opportunity cost. Finally, Python and the use of Bloomberg were essential for manipulating, controlling and visualizing the data.

Conversely, some subjects had to be deepened independently. I am thinking above all of data engineering and good development practices related to CI/CD, in particular automation, version control, testing and the reproducibility of processing, which are not really at the heart of the curriculum. I would also have liked to be trained more in data structures and algorithms (DSA), as well as in certain advanced quantitative methods such as stochastic control and optimal transport. These tools seem to me particularly useful for going further in quantitative finance, in particular in modern approaches to optimization, modeling and machine learning. These are therefore areas in which I intend to continue to progress.

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Appendices

Appendix A — Unconditional Correlation Matrix of the Factors

	Equities	Duration	Credit	Carry	Liquidity
Equities	1.00	0.55	−0.03	0.63	−0.07
Duration	0.55	1.00	−0.00	0.79	0.18
Credit	−0.03	−0.00	1.00	−0.31	−0.19
Carry	0.63	0.79	−0.31	1.00	0.22
Liquidity	−0.07	0.18	−0.19	0.22	1.00

Table A.1 — Unconditional correlations of the factors (2004–2026)

Appendix B — Distribution of Regimes

Regime	Number of months	Frequency
Goldilocks	121	45.1 %
Overheating	82	30.6 %
Recession	33	12.3 %
Stagflation	32	11.9 %
Total	268	100 %

Table B.1 — Distribution of observations by regime

Appendix C — Formulas and Mathematical Approach

A. Construction of the Factors

Simple monthly return of index i:

$$r(i,t) \; = \; P(i,t) \,/\, P(i,t-1) \; - \; 1$$

USD-denominated indices are converted into EUR: $P^{EUR} = P^{USD}/(EUR/USD)$

The five factors are:

$$Actions = r(MSCIWorld, EUR)$$
$$Duration = r(EuroTreasury)$$
$$Carry = r(EuroHighYield)$$

Credit, isolated from the rate effect by a duration-neutral residual (β = hedge ratio):

$$Crédit(t) = r_I G(t) - \beta \cdot r_{Duration}(t), \beta = Cov(r_{IG}, r_{Duration})/Var(r_{Duration})$$

Liquidity, illiquidity premium (long emerging markets / short developed markets):

$$Liquidité(t) \; = \; r(MSCI\ EM, EUR) \; - \; r(MSCI\ World, EUR)$$

B. Regime Detection (Hidden Markov Model)

Latent states s(t) ∈ {1,…,K}, observations x(t) = (growth, inflation), Gaussian emissions:

$$x(t)|s(t) = k \sim N(\mu_k, \Sigma_k)$$

Transition matrix A and emission probabilities estimated by the Baum-Welch algorithm (EM), which maximizes the log-likelihood L:

$$A(j,k) \; = \; P(s(t) = k \mid s(t-1) = j)$$

Filtered probabilities (forward recursion), then smoothed (forward-backward):

$$\pi_t(k) \; = \; P(s(t) = k \mid x(1{:}t))$$

C. Conditional Opportunity Cost

The opportunity cost of a factor i in regime k is approximated by its conditional Sharpe ratio (rf = 0), annualized:

$$SR_k(i) = \big(\mu_k(i)/\sigma_k(i)\big) \cdot \sqrt{1}2$$

μk(i) and σk(i) are estimated on the months classified in regime k. The instability of the ranking between regimes is measured by Spearman's rank correlation ρS.

D. Regime-Conditional Moments (TPA)

At each date t, the moments are weighted by the filtered regime probabilities:

$$\mu(t) = \Sigma_k \pi_t(k) \cdot \mu_k \qquad \Sigma(t) = \Sigma_k \pi_t(k) \cdot \Sigma_k$$

E. Portfolio Optimization

MVO – maximization of the Sharpe ratio:

$$max_w (w^{\mathrm{T}}\mu)/\sqrt{(w^{\mathrm{T}}\Sigma w)}$$

$$s.c. \quad \Sigma_i w_i = 1, \qquad w_{min} \leq w_i \leq w_{max}$$

ERC – equality of risk contributions RCi:

$$RC_i = w_i \cdot (\Sigma w)_i \qquad objectif: \quad min \Sigma_{i,j}\big(RC_i - RC_j\big)^2$$

SAA: fixed weights (calibrated in-sample). TPA: weights recomputed with (μ(t), Σ(t)) in walk-forward, annual recalibration. The HMM is re-estimated at each recalibration on past data only (expanding window) and the decision is lagged by one month, which eliminates any look-ahead bias in the regime detection.

F. Performance Metrics (Out-of-Sample)

The performances of the different allocation rules are compared using several classic indicators: annualized return, annualized volatility, the Sharpe ratio, the maximum drawdown and the monthly 95% CVaR.

The annualized Sharpe ratio is computed as follows:

$$Sharpe \;=\; (\,moyenne(r)\;/\;\acute{e}cart - type(r)\,)\;\cdot\;\sqrt{12}$$

The maximum drawdown measures the maximum loss observed relative to the historical peak of the portfolio value:

$$MaxDD = min_t[V_t/max_{s\le t}V_s - 1]$$

Finally, the 95% CVaR corresponds to the average loss observed in the worst 5% of months:

$$CVaR_{95\%}(mensuel) \;=\; E[\,r\,|\,r\;\le\;quantile_{5\%}(r)\,]$$

G. Jobson–Korkie Test (Memmel Correction)

To compare the Sharpe ratios obtained out-of-sample, we use the Jobson–Korkie test, with the correction proposed by Memmel. This test makes it possible to assess whether the gap between two Sharpe ratios is statistically significant.

For two monthly Sharpe ratios SR_1, SR_2 with correlation ρ_{12}, the test is written:

$$z \;=\; (SR_1 \;-\; SR_2)\;/\;\sqrt{\theta}$$

where θ depends on the sample size, on the two Sharpe ratios and on the correlation ρ12 between the returns of the two strategies:

$$\theta \;=\; (1/T)\;\cdot\;[\;2(1\;-\;\rho_{12})\;+\;½\,(\,SR_1{}^2\;+\;SR_2{}^2\;-\;2\cdot SR_1\cdot SR_2\cdot \rho_{12}{}^2\,)\,]$$

The two-sided p-value is then computed from the standard normal distribution.

H. Conditional Allocation Weights by Regime

This appendix details the weights produced by the conditional allocation in each of the four regimes. It is the allocation that would be retained if one were certain of being in the regime considered: the moments (μ_k, Σ_k) are estimated on the months classified in regime k, then the corresponding optimization program is solved (MVO max-Sharpe or ERC). In the walk-forward backtest, the effective allocation at each date is a combination of these weights, weighted by the filtered regime probabilities from the HMM and recomputed on an expanding window; the tables below therefore provide an illustrative "pure-state" reading. The values are expressed as percentages (long-only weights, summing to 100%).

Table H.1 — TPA-MVO weights by regime (%)

Regime	**Equities**	**Duration**	**Credit**	**Carry**	**Liquidity**
Goldilocks	14.0	21.7	50.0	9.2	5.0
Overheating	7.3	40.0	5.0	42.7	5.0
Stagflation	50.0	5.0	35.0	5.0	5.0
Recession	5.0	48.1	5.0	18.0	23.9

The TPA-MVO shifts clearly from one regime to another, which illustrates the mechanism of conditioning: Duration dominates in Recession (48%), Carry in Overheating (43%), while Credit and Equities carry the allocation in Goldilocks. In the Stagflation regime, where all conditional Sharpe ratios are negative, mean-variance optimization produces an uninformative corner solution (concentration on Equities); this effect is attenuated in the backtest by the probabilistic weighting of regimes and by the use of an expanding window, so that the weights actually applied are smoother than those in the table.

Table H.2 — TPA-ERC weights by regime (%)

Regime	**Equities**	**Duration**	**Credit**	**Carry**	**Liquidity**
Goldilocks	15.2	45.2	1.0	23.0	15.6
Overheating	15.0	38.3	1.0	23.7	22.0
Stagflation	16.9	35.4	1.0	16.5	30.2
Recession	18.0	34.1	1.0	13.8	33.1

The TPA-ERC, which exploits only the covariance matrix, produces weights that are almost stable from one regime to another. The Credit factor, of very low volatility, systematically reaches its minimum bound (1%): at this level of volatility, it would take an unrealistic weight for it to contribute as much as the others to total risk, so that the ERC in practice bears on the four significantly risky factors. This insensitivity to the regime explains why the TPA-ERC does not improve on the SAA-

ERC: the gain from conditioning comes exclusively from the expected-return channel (μ), exploited by mean-variance, and not from the covariance structure.

I. Detailed Risk Indicators (Out-of-Sample)

This appendix complements Table 6.4 with a broader set of risk indicators, computed over the out-of-sample period (January 2021 – May 2026, 65 months). Table I.1 presents the absolute-risk measures; Table I.2 the relative-risk measures, the reference being the mean-variance SAA (SAA-MVO). The Calmar ratio relates the annualized return to the maximum drawdown; the tracking error is the annualized standard deviation of the active performance (portfolio − reference); the information ratio relates the average active performance to the tracking error; the beta and the correlation are estimated relative to the SAA-MVO.

Table I.1 — Absolute-risk indicators (OOS 2021–2026)

Strategy	Ann. return	Ann. vol.	Sharpe	Max DD	Calmar	CVaR 95%	% pos. months
SAA-MVO	0.9 %	3.6 %	0.25	−10.8 %	0.08	−2.4 %	55 %
SAA-ERC	1.7 %	5.2 %	0.35	−15.1 %	0.11	−3.4 %	54 %
TPA-MVO	2.5 %	3.8 %	0.66	−8.1 %	0.30	−1.9 %	62 %
TPA-ERC	1.4 %	5.3 %	0.29	−15.1 %	0.09	−3.4 %	54 %

Table I.2 — Relative-risk indicators versus the SAA-MVO (OOS)

Strategy	Tracking error	Information ratio	Beta	Correlation
SAA-MVO (ref.)	—	—	1.00	1.00
SAA-ERC	2.4 %	0.37	1.33	0.91
TPA-MVO	1.7 %	0.94	0.94	0.90
TPA-ERC	2.2 %	0.28	1.37	0.94

These indicators confirm and refine the central result. The conditional TPA-MVO dominates across all dimensions: the highest Sharpe ratio (0.66), the lowest maximum drawdown (−8.1%), the best Calmar ratio (0.30) and the highest proportion of positive months (62%). Above all, its information ratio versus the SAA-MVO reaches 0.94 for a moderate tracking error (1.7%), which reflects an efficient active outperformance and not a mere increase in risk: its beta is below 1 (0.94). Conversely, the ERC allocations show higher drawdowns (−15.1%) and betas above 1, consistent with their overweighting of the bond factors penalized in 2022.

J. Opportunity Cost Net of Regulatory Capital (Solvency II)

This appendix proposes a reading of the opportunity cost net of the Solvency II capital charge. Each factor is associated with a stylized SCR, inspired by the shocks of the standard formula (type-1 equities: 39%; emerging / type-2 equities: 49%; credit spread and high yield according to duration and quality; interest-rate risk for Duration). The cost of capital is set at 6% (consistent with the risk margin). The net return subtracts this cost of capital from the annualized return, and the net Sharpe ratio follows. The SCR values are deliberately approximate and illustrative: the objective is to show the distortion of the ranking, not an exact regulatory calculation.

Table J.1 — Gross and net-of-capital opportunity cost (stylized SCR, cost of capital 6%)

Factor	**Ann. return**	**Ann. vol.**	**SCR**	**Capital cost**	**Net return**	**Gross Sharpe**	**Net Sharpe**
Equities	9.9 %	13.1 %	39 %	2.34 %	7.6 %	0.76	0.58
Duration	3.6 %	4.8 %	8 %	0.48 %	3.2 %	0.76	0.66
Credit	−0.1 %	1.1 %	15 %	0.90 %	−1.0 %	−0.06	−0.85
Carry	6.9 %	9.6 %	35 %	2.10 %	4.8 %	0.72	0.50
Liquidity	−2.1 %	12.0 %	49 %	2.94 %	−5.0 %	−0.17	−0.42

Net of capital, the ranking moves from “Duration ≈ Equities > Carry > Credit > Liquidity” to “Duration > Equities > Carry > Liquidity > Credit”: the capital charge breaks the tie at the top in favor of Duration, which is not capital-hungry. Combined with the regime-conditional reading (Duration already dominates in Recession), this penalty reinforces an insurer's preference for rate factors and illustrates why an insurer's opportunity cost cannot be reduced to the return-risk trade-off alone.